DSP – Practice Problem Set #2

1. For the following CT signals, calculate the maximum sampling period T_s that produces no aliasing:
   (a) $x_1(t) = 5\,\text{sinc}(200t)$;
   (b) $x_2(t) = 5\,\text{sinc}(200t) + 8\sin(100\pi t)$;
   (c) $x_3(t) = 5\,\text{sinc}(200t)\sin(100\pi t)$;
   (d) $x_4(t) = 5\,\text{sinc}(200t) * \sin(100\pi t)$, where $*$ denotes the CT convolution operation.

2. The CT signal $x(t) = \sin(400\pi t) + 2\cos(150\pi t)$ is sampled with an ideal impulse train. Sketch the CTFT of the sampled signal for the following values of the sampling rate:
   (a) $f_s = 100$ samples/s;
   (b) $f_s = 200$ samples/s;
   (c) $f_s = 400$ samples/s;
   (d) $f_s = 500$ samples/s.
   In each case, calculate the reconstructed signal using an ideal LPF with the transfer function given in Eq. (9.7) and a cut-off frequency of $\omega_s/2 = \pi f_s$.

3. A CT band-limited signal $x(t)$ is sampled at its Nyquist rate f_s and transmitted over a band-limited channel modeled with the transfer function

$$H_{ch}(\omega) = \begin{cases} 1 & 4\pi f_s \le |\omega| \le 8\pi f_s \\ 0 & \text{otherwise.} \end{cases}$$

   Let the signal received at the end of the channel be $x_{ch}(t)$. Determine the reconstruction system that recovers the CT signal $x(t)$ from $x_{ch}(t)$.

4. Consider a digital mp3 player that has 1024×10^6 bytes of memory. Assume that the audio clips stored in the player have an average duration of five minutes.

(a) Assuming a sampling rate of 44 100 samples/s and 16 bits/sample/channel quantization, determine the average storage space required (without any form of compression) to store a stereo (i.e. two-channel) audio clip.

(b) Assume that the audio clips are stored in the mp3 format, which reduces the audio file size to roughly one-eighth of its original size. Calculate the storage space required to store an mp3-compressed audio clip.

(c) How many mp3-compressed audio files can be stored in the mp3 player?

5. Consider the input sequence $x[k] = 2u[k]$ applied to a DT system modeled with the following input–output relationship:

$$y[k+1] - 2y[k] = x[k],$$

and ancillary condition $y[-1] = 2$.

(a) Determine the response $y[k]$ by iterating the difference equation for $0 \le k \le 5$.

(b) Determine the zero-state response $y_{\text{zi}}[k]$ for $0 \le k \le 5$.

(c) Calculate the zero-input response $y_{\text{zs}}[k]$ for $0 \le k \le 5$.

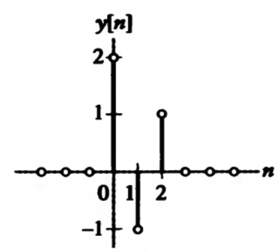
(d) Verify that $y[k] = y_{\text{zi}}[k] + y_{\text{zs}}[k]$.

6. **A discrete-time system is both linear and time invariant. Suppose the output due to an input $x[n] = \delta[n]$ is given in Fig. P1.77(a).**

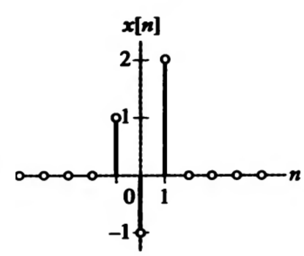
(a) Find the output due to an input $x[n] = \delta[n-1]$.

(b) Find the output due to an input $x[n] = 2\delta[n] - \delta[n-2]$.

(c) Find the output due to the input depicted in Fig. P1.77(b).



(a)



(b)

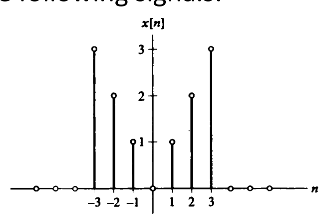
7. For each of the following impulse responses, determine whether the corresponding system is (i) causal, and (ii) stable.

$h[n] = (-1)^n u[-n]$
$h[n] = (1/2)^{|n|}$
$h[n] = \cos\left(\frac{\pi}{8}n\right)\{u[n] - u[n-10]\}$
$h[n] = 2u[n] - 2u[n-5]$
$h[n] = \sin\left(\frac{\pi}{2}n\right)$
$h[n] = \sum_{p=-1}^{\infty} \delta[n-2p]$

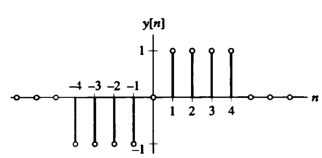
8. Determine the output of the systems described by the following difference equations with input and initial conditions as specified:
   (a) $y[n] - \frac{1}{2}y[n-1] = 2x[n]$,
   $y[-1] = 3, x[n] = \left(\frac{-1}{2}\right)^n u[n]$
   (b) $y[n] - \frac{1}{9}y[n-2] = x[n-1]$,
   $y[-1] = 1, y[-2] = 0, x[n] = u[n]$
   (c) $y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$,
   $y[-1] = 4, y[-2] = -2, x[n] = (-1)^n u[n]$
   (d) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$,
   $y[-1] = 1, y[-2] = -1, x[n] = 2u[n]$

9. The systems that follow have input x[n] and output y[n]. For each system, determine whether it is (i) linear, (ii) time-invariant, (iii) stable, and (iv) causal:
$y[n] = 2x[n]u[n]$
$y[n] = \log_{10}(|x[n]|)$
$y[n] = \sum_{k=-\infty}^{n} x[k+2]$
$y[n] = \cos(2\pi x[n+1]) + x[n]$
$y[n] = x[n]\sum_{k=-\infty}^{\infty} \delta[n-2k]$
$y[n] = 2x[2^n]$

10. Given x[n] and y[n] as shown, sketch the following signals:
   (a) $x[2n]$
   (b) $x[3n-1]$
   (c) $y[1-n]$
   (d) $y[2-2n]$
   (e) $x[n-2] + y[n+2]$
   (f) $x[2n] + y[n-4]$
   (g) $x[n+2]y[n-2]$
   (h) $x[3-n]y[n]$
   (i) $x[-n]y[-n]$
   (j) $x[n]y[-2-n]$
   (k) $x[n+2]y[6-n]$



(a)



(b)

11. Determine whether the following signals are periodic, and for those which are, find the fundamental period:

$x[n] = \cos\left(\frac{8}{15}\pi n\right)$
$x[n] = \cos\left(\frac{7}{15}\pi n\right)$
$x[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-3k] + \delta[n-k^2]\}$
$x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$
$x[n] = (-1)^n$
$x[n] = (-1)^{n^2}$

$x[n] = \cos(2n)$
$x[n] = \cos(2\pi n)$