Multi Resolution Bilateral Filter for MR Image Denoising

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Abstract— **Clinical magnetic resonance imaging (MRI) data is normally corrupted by random noise from the measurement process which reduces the accuracy and reliability of any automatic analysis. For this reason, denoising methods are often applied to increase the : Signal-to-Noise Ratio (SNR) and improve image quality. The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. In spite of the sophistication of the recently proposed methods, most algorithms have not yet attained a desirable level of applicability. All show an outstanding performance when the image model corresponds to the algorithm assumptions but fail in general and create artifacts or remove image fine structures. In this paper we propose an extension of the bilateral filter: multi resolution bilateral filter (MRBF), with wavelet transform (WT) sub-bands mixing. The proposed wavelet sub-bands mixing is based on a multi resolution approach for improving the quality of image denoising filter, which turns out to be very effective in eliminating noise in noisy images. Quantitative validation was carried out on synthetic datasets generated with the Brain Web simulator. Comparison with other methods, such as nonlinear diffusion, Fourth-Order Partial Differential Equations , Total variation, Nonlocal mean, Wavelet thresholding, and Bilateral filters , shows that the proposed multi resolution bilateral filter (MRBF) produces better denoising results. The mathematical analysis is based on the analysis of the "method noise", defined as the difference between a digital image and its denoised version. The MRBF algorithm is also proven to be asymptotically optimal under a generic statistical image model. The most powerful evaluation method seems, however, to be the visualization of the method noise on natural images. The more this method noise looks like a real white noise, the better the method**.

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Keywords- magnetic resonance imaging, denoising algorithms, multi resolution, method noise, wavelet transform, bilateral filter, image quality factor.

I. INTRODUCTION

Image denoising can be considered as a component of processing or as a process itself. In the first case, the image denoising is used to improve the accuracy of various image processing algorithms such as registration or segmentation.. Then, the quality of the artifact correction influences performance of the procedure. In the second case, the noise removal aims at improving the image quality for visual inspection.. The preservation of relevant image information is important, especially in a medical context. This paper focuses on a new denoising method firstly introduced by Tomasi et al. [1] for 2D image denoising: the bilateral filter. We propose, to improve this filter with an automatic tuning of the filtering parameter, and a mixing of wavelet su-bands based on the approach proposed in [2]. These contributions lead to a fully-automated method and overcome the main limitation of the classical Bilateral filter.

Many denoising methods have been developed over the years; among these methods, wavelet thresholding is one of the most popular approaches. In wavelet thresholding, a signal is decomposed into its approximation (low-frequency) and detail (high-frequency) sub-bands; since most of the image information is concentrated in a few large coefficients, the detail s sub-bands are processed with hard or soft thresholding operations [3].

A recently popular denosing method is the bilateral filter [1].The bilateral filter takes a weighted sum of the pixels in a local neighborhood; the weights depend on both the spatial distance and the intensity distance. In this way, edges are preserved well while noise is averaged out. Mathematically,

$$\tilde{I}(X)=\frac{1}{C}\sum_{y\in N(X)} e^{\frac{-\|y-x\|^2}{2\partial_d^2}}\, e^{\frac{-|I(y)-I(x)|^2}{2\partial_r^2}}\, I(y)\,, \tag{1}$$

where σ_d and σ_r are parameters controlling the fall-off of the weights in spatial and intensity domains, respectively, *N(x)* is a spatial neighborhood of *I(x)*, and *C* is the normalization constant:

$$C=\sum_{y\in N(x)} e^{\frac{-\|y-x\|^2}{2\partial_d^2}}\, e^{\frac{-|I(y)-I(x)|^2}{2\sigma_r^2}}\ . \tag{2}$$

Although the bilateral filter was first proposed as an intuitive tool, recent papers have pointed out the connections with some other techniques. In [4], it is shown that the bilateral filter is identical to the first iteration of the Jacobi algorithm (diagonal normalized steepest descent) with a specific cost function. [5-6] relate the bilateral filter with the anisotropic diffusion. The bilateral filter can also be viewed as an Euclidean approximation of the Beltrami flow, which produces a spectrum of image enhancement algorithms ranging from the L2 linear diffusion to the L1 nonlinear flows [7], [8], [9]. In [6], it is shown that nonlocal means filter,

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where similarity of local patches is used in determining the pixel weights. When the patch size is reduced to one pixel, the nonlocal means filter becomes equivalent to the bilateral filter. [10] extends the work of [3] by controlling the neighborhood of each pixel adaptively.

II. NOISE PROPERTIES OF MR DATA

In MRI, the acquired complex data is known to be polluted by white noise which is characterized by a Gaussian PDF. After inverse Fourier transformation, the real and imaginary data is still corrupted with Gaussian noise because of the orthogonality of the Fourier transform. However, it is common practice to transform the complex valued images into magnitude and phase images [11-12].

A. Noise model

The magnitude images are formed by calculating the magnitude, pixel by pixel, from the real and the imaginary images. This is a nonlinear mapping and therefore the noise distribution is no longer Gaussian.

The image pixel intensity in the absence of noise is denoted by *A* and the measured pixel intensity by *M*. In the presence of noise, the probability distribution for *M* can be shown to be given by [13].

$$P_M(M) = \frac{M}{\sigma^2} e^{-(M^2+A^2)}/2\sigma^2 \, I_O\left(\frac{A.M}{\sigma^2}\right) . \qquad (3)$$

where I_0 is the modified zeroth order Bessel function of the first kind and σ denotes the standard deviation of the Gaussian noise in the real and the imaginary images (which we assume to be equal). This is known as the *Rice* density. As can be seen the Rician distribution is far from being Gaussian for small SNR ($A/\sigma \leq 1$). For ratios as small as $A/\sigma = 3$, however, it starts to approximate the Gaussian distribution.

A special case of the Rician distribution is obtained in image regions where only noise is present, $A = 0$. This is better known as the *Rayleigh* distribution and Eq. (3) reduces to

$$P_M(M) = \frac{\frac{M}{\sigma^2} e^{-(M^2)}}{2\sigma^2} . \qquad (4)$$

This Rayleigh distribution governs the noise in image regions with no NMR signal. The mean and the variance for this distribution can be evaluated analytically and are given by [11].

$$M = \sigma\sqrt{\pi/2} \, and \; \sigma_M^2 = (2-\pi/2)\sigma^2 . \qquad (5)$$

These relations can be used to estimate the "true" noise power, σ^2 ,from the magnitude image. Another interesting limit of Eq. (3) is when the SNR is large.

$$P_M(M) \approx \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1(M-\sqrt{A^2}+\sigma^2)^2/2\sigma^2} . \qquad (6)$$

This equation shows that for image regions with large signal intensities the noise distribution can be considered as a Gaussian distribution with variance σ^2 and mean $\sqrt{A^2+\sigma^2}$. [13-14].

III. METHODOLOGY

A. *Multi Resolution Bilateral Filter*

Image noise is not necessarily white and may have different spatial frequency (fine-grain and coarse-grain) characteristics. Multi resolution analysis has been proven to be an important tool for eliminating noise in signals; it is possible to distinguish between noise and image information better at one resolution level than another [5]. Therefore, we decided to put the bilateral filter in a multi resolution framework: Referring to Figure 1, (i) Denoising of the original image *I* using two sets of filtering parameters. This yields two images *Io* and *Iu*. In *Io*, the noise is efficiently removed and, conversely, in *Iu*, the image features are preserved. (ii) Decomposing *Io* and *Iu* into low- and high-frequency sub-bands. The first level decomposition of the images is performed with a wavelet transform (WT). (iii) Mixing the highest-frequency sub-bands of *Io* and the lowest frequency sub-bands of *Iu*. (iv) Reconstructing the final image by an inverse WT from the combination of the selected high and low frequencies.

1. 1. Selection of wavelet sub-bands

Once the original image *I* has been denoised using two sets of filtering parameters, a WT at the first level is performed on both *Io* and *Iu* images. For each image, eight sub-bands are obtained: LLL1, LLH1, LHL1, HLL1, LHH1, HLH1, HHL1, and HHH1.

(i) In the eight wavelet s sub-bands obtained with *Io*, the frequencies corresponding to noise are efficiently removed from the high frequencies whereas the low frequencies associated to the main features are spoiled. The experiment results will be given in the next section (ii) In the eight wavelet s sub-bands obtained with *Iu*, the low frequencies associated to main features are efficiently preserved whereas residual frequencies corresponding to noise are present in high frequencies.

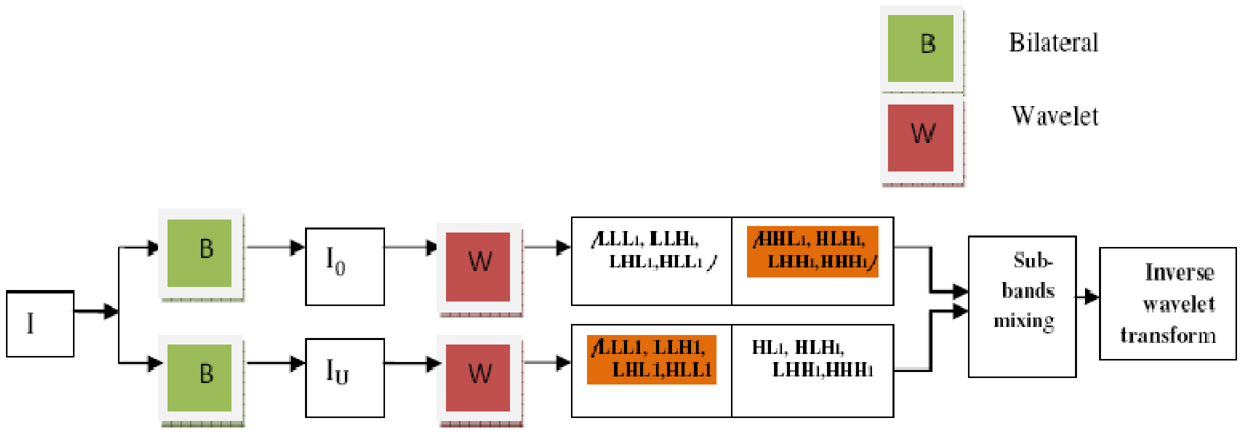

Figure 1.. First, the noisy image *I* is denoised with two sets of filtering parameters I_u and I_o. Then, I_u and I_o are decomposed into low- and high-frequency sub-bands by WT. The four lowest frequency sub-bands of *Iu* (i.e., LLL1, LLH1, LHL1, and HLL1) are mixed with the four highest-frequency sub-bands of *Io* (i.e., LHH1, HLH1, HHL1, and HHH1). Finally, the result image is obtained by inverse WT of the selected sub-bands

B. The Method noise

Definition 1.1 (method noise). *Let u be a (not necessarily noisy) image and* D_h *a denoising operator depending on h. Then we define the* method noise *of u as the image difference.*

$$(D_h, u) = u - D_h\,(u)\,. \tag{7}$$

This method noise should be as similar to a white noise as possible. In addition, since we would like the original image u not to be altered by denoising methods, the method noise should be as small as possible for the functions with the right regularity [3].

According to the preceding discussion, four criteria can and will be taken into account in the comparison of denoising methods:

- A display of typical artifacts in denoised images.
- A formal computation of the method noise on smooth images, evaluating how small it is in accordance with image local smoothness.
- A comparative display of the *method noise* of each method on real images with $\sigma = 5$. We mentioned that a noise standard deviation smaller than 6 is subliminal, and it is expected that most digitization methods allow themselves this kind of noise.
- A classical comparison receipt based on noise simulation: it consists of taking a good quality image, adding Gaussian white noise with known σ, and then computing the best image recovered from the noisy one by each method.

IV. RESULTS AND DISCUSSION

To conduct the experiments over synthetic data simulated MR image (T1) with 1 voxel resolution (8 bit quantization) from the Brain web phantom [16] were used. We added zero mean Gaussian noise to the simulated MR data Fig.2. All experiments were performed using MATLAB 7.0 (Mathworks Inc.).

We have compared, qualitatively and quantitatively, the performance of our proposed algorithm with six state-of-the-art filtering algorithms, the Anisotropic Diffusion filter (ADF), Fourth-Order Partial Differential Equations (4^{th} PDE), Total variation(TV), Nonlocal mean(NLM),Wavelet thresholding, and Bilateral Filtering algorithms.

The following set of figures show the results of applying denoising methods.

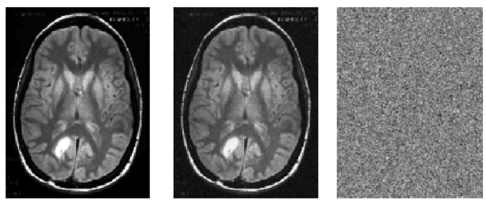

Fig. 2.From left to right: T1-weighted synthetic noise-free MR image, the corresponding noisy image (σ = 5) and applied the methods noise (difference between both images) .

To measure the quality of the filters the following TABLE 1 shows the Signal-to-Noise Ratio (SNR), Peak Signal-to-Noise Ratio (PSNR) ,and Root Mean Squared Error (RMSE). Best values obtained for *Multi Resolution Bilateral* , *Nonlocal mean* ,*Fourth-Order Partial Differential Equations* , *Total variation*, , and *Bilateral filters* with lower RMSE, and higher SNR, and PSNR.

A. The Method noise comparison

In previous section we have defined the method noise and computed it for the different algorithms. Remember that the denoising algorithm is applied on the original (slightly noisy) image. A filtering parameter, depending mainly on the standard deviation of the noise, must be fixed for the most part

TABLE 1 Image Quality Evaluation Metrics at Statistical Measurements; for Different Filter Types

Filter types	Feature set		
	SNR	*PSNR*	*RMSE*
4^{th}-order	**42.0746**	**51.8437**	**4.6145**
Anisotropic	**41.9855**	**51.6090**	11.1376
Wavelet	39.0936	**51.7173**	89.7410
TV	**42.1032**	**51.1411**	**4.8768**
Bilateral	**42.1040**	**51.8437**	**5.6657**
NLM	**42.1845**	**52.6756**	**3.5779**
MRBF	**42.2069**	**52.9729**	**2.2848**

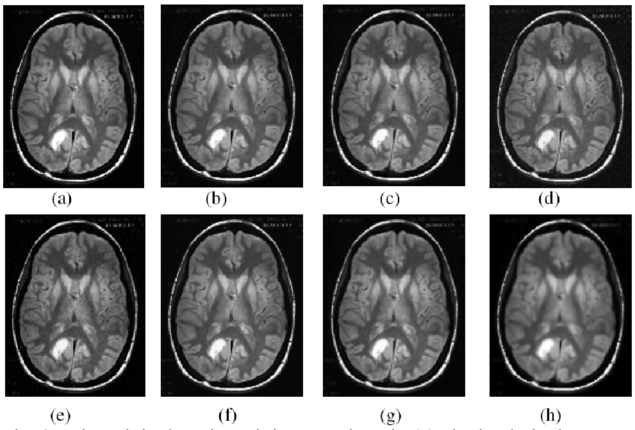

Fig. 3. The original Brain web images given in (a) obtained via the compared methods given in(b)-(k).(a) Original image. (b) ADF.(c) 4^{th} -order PDE.(d)TV. (e) Bilateral filters.(f) MRBF . (g) NLM .(h) Wavelet.

of algorithms. Let us σ = 5: we can suppose that any image is affected by this amount of noise since it is not visually noticeable. The method noise tells us which geometrical features or details are preserved by the denoising process and which are eliminated.

From Fig.4. Let us comment on denoising methods briefly.

- The *anisotropic filter* method noise displays the corners and high frequency features, but suffer in the presence of texture.
- The *Total Variation* method modifies most structures and details of the image.
- The *Nonlocal mean* method noise reduces the loss of destroying details, and removed noise.
- In general, *wavelet thresholding* gave poorer performance for removing the noise from MR image .
- *Bilateral filtering* smoothes images while preserving edges.
- The *Fourth-Order Partial Differential Equations* try to reduce the geometry present in the removed noise, adding it back to the restored image, and therefore reducing the method noise.
- *The Multi Resolution Bilateral* method noise looks the more like white noise.

Note that, in presence of periodic or stochastic patterns, *multi resolution bilateral filter* square error PSNR and SNR are significantly more precise than the other algorithms.

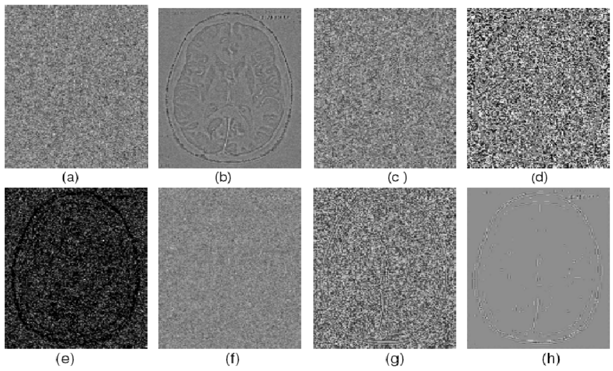

Fig. 4. Image method noise: (a) Gaussian white noise .(b) ADF(c) MRBF. (d) TV. (e) Bilateral. (f) NLM. (g) 4^{th} -order PDE.(h) Wavelet.

V. CONCLUSION

In this work we present a multi resolution image denoising framework, which integrates bilateral filtering and wavelet transform. In this framework, we decompose an image into low- and high-frequency components, and apply wavelet sub-bands mixing .We have discussed the different approaches which resort to suitable image denoising algorithms and the best techniques found were multi resolution bilateral filter. This shows the promising results in produce accurate result than previous methods, the multi resolution bilateral algorithm seems to denoise the image, keeping the main structures and details. The comparison with well established methods such as NLM filter, nonlinear diffusion, Fourth-Order Partial Differential Equations, Wavelet thresholding, Bilateral, and TV minimization shows that the multi resolution bilateral filter produces better results. Finally, the impact of the proposed multi resolution approach based on wavelet sub-bands mixing should be investigated further, for instance, when combined to the nonlinear diffusion filter [17] and the total variation minimization [18].

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