Study of Compressed Sensing Application to Low Dose Computed Tomography Data Collection

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***Abstract*— Computed Tomography (CT) is one of the most used imaging techniques in medical field which enables us to generate two-dimensional as well as three-dimensional image of the inside the body. Fundamentally, any imaging system contains two main stages the data collection and the image reconstruction. For CT, The data collection stage depends on the resolution of image collected and field of view. The image reconstruction of an object depends on projections by passing a series of rays through an object. The problem in general is that CT is computationally very intensive, due to the large number of projections. The large computational requirements have led to large times for CT image reconstruction, and extra X-ray dose to get high quality images. To accelerate the reconstruction process and decrease the effect of x-ray on patients, we need to decrease the number of projections. In this paper we introduce a compressed sensing (CS) technique for CT to reconstruct images from reduced projection data and compare it with other algorithms, A CS iterative algorithm reconstructed an image of digital Shepp-Logan phantom using small number of projections with high quality resolution compared to traditional iterative technique, we studied the effect of algorithm controlling parameters on the reconstructed image. Finally using the proposed technique will decrease the high risk associated with the high dose x-ray needed in the traditional CT scans.**

***Keywords*— computed tomography; compressed sensing, projections; reconstruction.**

I. Introduction

Computed tomography (CT) imaging is one of the most important imaging devices that create detailed two-dimensional cross-sectional images from three-dimensional body structures by using rotating x-ray equipment, combined with a digital computer. Fundamentally, tomography deals with a series of projection data of objects from several view angles, and gets the reconstructed image from the mathematical solution of the acquired projections [1].

After collecting projections' data of the scanned body, we need to reconstruct these projections into an actual image, this operation is called CT image reconstruction which is creating a 2-D or 3-D image from radiation readings acquired from the CT imaging operation, for example we can generate a three-dimensional image of the body from a series of individual camera images. To reconstruct an image, some mathematical operations must be done to generate the image and to make some processing on it like sharpening or to remove blurring to be clear and useful enough [2].

We have two methods to solve the reconstruction problem: either the analytical method or the algebraic method[3]. The analytical method, such as filtered back projection (FBP) method, are relatively high computational speed and short computational time, but it needs many projection data to reconstruct images accurately [4], but its main drawback is that the analytic algorithms will reconstruct images with severe aliasing artifacts for a reduced number of projections [5]. The iterative algorithms such as Algebraic reconstruction technique (ART), on the contrary, can reconstruct images from relatively less projection data than the analytical algorithms. But, it will take much longer time that is why we use the iterative algorithm in our technique.

Compressed sensing theory (CS) [6] states that one can recover certain signals and images from far fewer samples or measurements than traditional methods use. CS depends on two principles: sparsity and incoherence [7]. Sparsity expresses the idea that the "information rate" of a continuous time signal may be much smaller than suggested by its bandwidth, or that a discrete-time signal depends on a number of degrees of freedom which is comparably much smaller than its (finite) length. More precisely, CS exploits the fact that many natural signals are sparse or compressible in the sense that they have concise representations when expressed in the proper basis. Incoherence extends the duality between time and frequency and expresses the idea that objects having a sparse representation in basis must be spread out in the domain in which they are acquired, incoherence says that the under sampling criterion should give noise like artifacts in the sparsifying domain [8].

In biomedical applications, scanning time and radiation dose are the most two important parameters in CT Scans, to reduce radiation dose and shorten scanning time, we need to decrease the number of projections or speed up the time needed at each projection, decreasing the time of projections leads to damage the biological specimens, and if we apply FBP or ART with limited number of projections we couldn't reconstruct image with high resolution as both algorithms mainly requires large number of projections. To solve this problem either to interpolate or extrapolate the missing data from the measured data then apply analytical reconstruction, this method couldn't give general conclusion from it, the other way to apply iterative algorithms to solve the problem from the insufficient data, this techniques are differ in the constraints[8]. In this paper we use the ART iterative technique and Sparsity constraint to solve this problem.

II. Materials and Methods

Inspired by the CS theory's success in signal recovery, we have anticipated that a CS-based algorithm may be used to reconstruct images from substantially reduced projection data. The algorithm minimizes the L1-norm of the sparse image as the constraint factor for the sparsity condition. This work focuses on reconstructing images from significantly reduced projection data and minimizing radiation dose without reducing image quality.

A successful application of CS requires that the desired image should have a sparse representation in a known transform domain [6]. Consider an image f, which can be viewed as an N × 1 column vector in R^N, whose individual elements f_j, j = 1, 2, .. N are N pixel values of the image. Expand vector f in an orthonormal basis ψ as Eq. (1).

$$f = \psi X. \tag{1}$$

Here f is the image, ψ is the N by N matrix $[\psi_1 ..., \psi_N]$ and X is also an N × 1 column vector.

If all but a few of entries in vector X are zero or almost zero, we will say that f is sparse in the ψ domain and X is its sparse representation.

For example, the Shepp-Logan phantom 256x256 pixels in fig.(1a) and its gradient counterpart in fig.(1b), The number of non-zero pixels in (fig.(1a)) is 27521which is about 42% of the total pixels

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number, while the number of non-zero pixels in its gradient image (fig.(1b)) is only 2182 which is about 3.3% of the total pixels number, which is much less than the non-zero pixel number of the original image. That means clearly that ψ domain is said to be the sparse domain of the image. Incoherence depends on the sampling pattern, the samples couldn't be uniform as it will cause aliasing , but applying random uniform sampling will result in noise in the reconstructed image which could be removed using L1 constraint [6].

As Sparsity and Incoherence can be applied for our case, we can reconstruct an image from low projections and apply CS algorithm.

In this algorithm to reconstruct image using CS we need to have initial reconstructed image and the sparse domain, we consider ART and L1 norm of the sparse image in gradient domain as the constraints of CS iterative algorithm. We test algorithm on phantom image using `phantom(128)` command from Matlab, we need to compute the attenuation coefficient of CT image which will be represented as the pixels of the image, to start the algorithm we need to compute the projections using parallel beam orientation and fan beam orientation as shown in fig.(2), the projection data is known as Sinogram [9] where the horizontal and vertical axes represent the detector-bin and view-angle coordinates, fig.(3) shows Shepp-Logan phantom Sinogram for parallel and fan beam orientations.

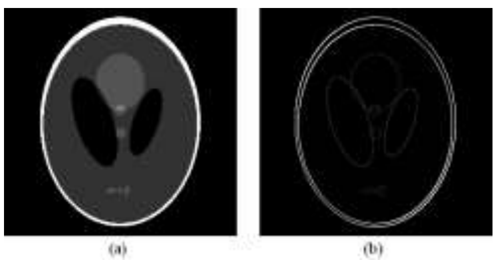

Figure 1: Shepp-Logan phantom. (a) The original image. (b) The gradient counterpart of (a).

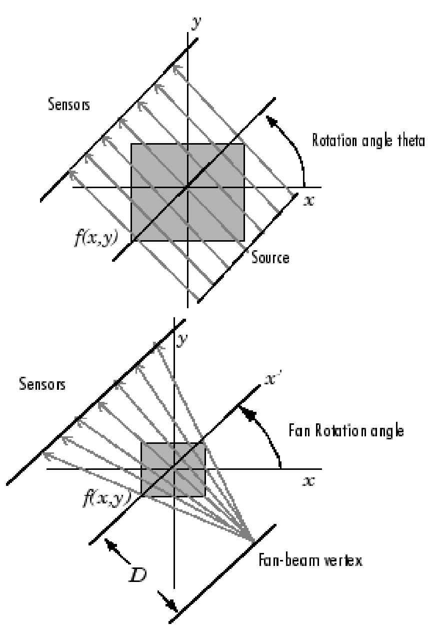

Figure 2: parallel and fan beam projections

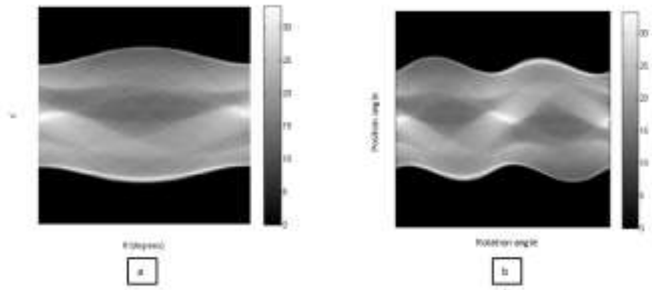

Figure 3: Shepp Logan phantom Sinogram for: a) parallel beam b) fan beam.

A. The Algorithm Outlines

Consider an image F (Nx1), suppose parallel-beam projection data of image F modeled by Eq. (2) [10].

$$P_i = \sum_{j=1}^{N} \emptyset_{i,j} F_j \ . \tag{2}$$

Here P_i is the projection ray, F is the image matrix and $\emptyset_{i,\,j}$ is the weight component of the intersection with matrix F which can be computed by the intersection length of the i[th] ray through the j[th] pixel image as shown in fig. (4).

To reconstruct an image it we use ART algorithm using eq. (3) [11].

$$x^{next} = x^{current} - backproject_{ray}\left\{\frac{backproject_{ray}(x^{current}) - measurment_{ray}}{normalization\ factor}\right\} \tag{3}$$

Here x is the pixel value, x_{next} is the pixel new value, $x_{current}$ is the pixel last value, $backproject_{ray}(x_{current})$ is the projection value using $x_{current}$, $measurement_{ray}$ is the true projection value, the normalization factor can be calculated depending on the pixels and projection intersection.

ART will not give accurate reconstructed image as the number of projections is less than the number of attenuation coefficients in the image [[12]] so we will move to the other constraint related to the L1 norm of the gradient image, this constrain aims to find the image F in gradient domain which satisfy Eq. (4) [11].

$$F = \arg min \left\|\tilde{F}\right\|_{L1} . \tag{4}$$

Where $\tilde{F}$ is the gradient image vector and L1 norm of $\tilde{F}$ ($\left\|\tilde{F}\right\|_{L1}$) $=\sum_{i=1}^{N}\left\|\tilde{F}_i\right\|$

To minimize L1 norm we can use the gradient descent method shown in eq. (5) [13].

$$F_{next} = F_{current} - \alpha\, \Delta_{current} \ . \tag{5}$$

Here F is the image, α is the gradient descent step which could be fixed number or variable number related to the error between iterations, we use many values shown in results method and Δ is the direction of L1 derivative of the gradient image.

The derivative of the L1 norm for the gradient descent algorithm could be calculated by numerical solution which can be expressed as the partial derivative [7][10] [15] [17].

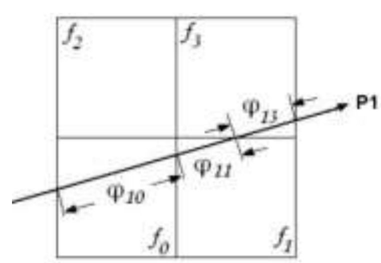

Figure 4: One projection and its intersection weighted matrix

B. *The Algorithm steps*

We use the algorithm shown in Fig.(6) to reconstruct the Shepp-Logan phantom images.

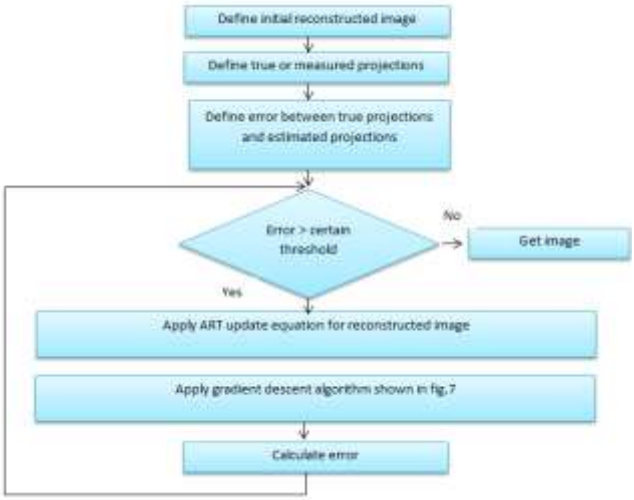

Figure 5: Proposed algorithm flowchart.

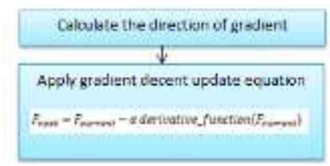

Figure 6: Gradient descent algorithm.

III. RESULTS

We use CS iterative algorithm to reconstruct a Shepp Logan phantom with different controlling parameters using Matlab program.

We will show the effect of controlling parameters on the reconstructed image and the relative error in each case, the root mean square error (RMSE) and the relative error are defined [16] as shown in Eq. (6).

$$RMSE = \sqrt{\frac{(F_r - F_0)^2}{N}}, \qquad Relative\ error = \frac{RMSE}{\overline{F_0}}. \tag{6}$$

Here F_r is the reconstructed image, F_0 is the true image; N is the number of image pixels and $\overline{F_0}$ is the mean value of pixels defined as $\frac{1}{N}\sum_{i=1}^{N} F_{0i}$.

A. *ART Versus CS Reconstruction Algorithms*

We used under-sampled projection data in view angle. We performed traditional ART and CS iterative reconstruction algorithms for 60 view angles using parallel and fan beam projections. Reconstructed images for parallel beam projections are shown in Fig.(7) and its relative error shown in Fig.(8) and images for fan beam are shown in Fig.(9) and its relative error shown in Fig.(10). Images reconstructed using CS algorithms have better resolution without noise as using traditional ART.

Figure 7: Reconstructed image using ART and CS for parallel beam projections.

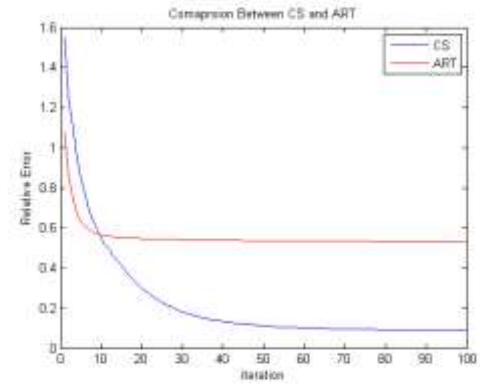

Figure 8: Relative Error of ART versus CS for parallel beam projections.

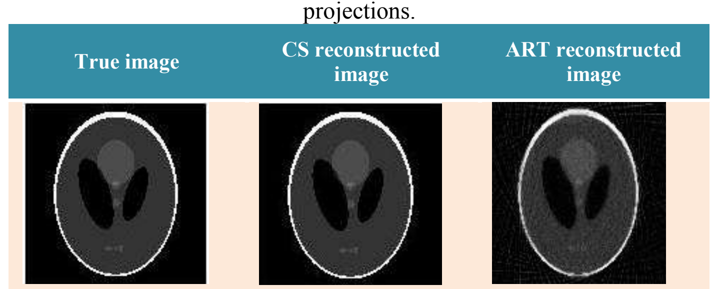

Figure 9: Reconstructed image using ART and CS for fan beam projections.

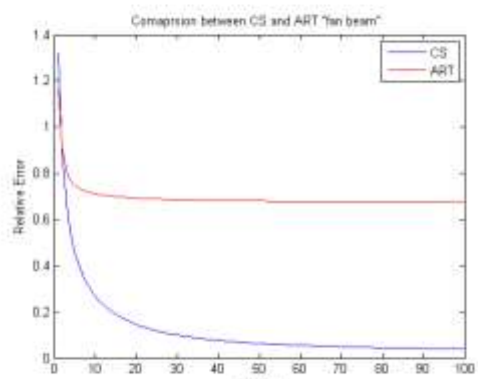

Figure 10: Relative Error of ART versus CS for fan beam projections.

B. *Controlling Parameters*

There are some controlling parameters affects the quality of the reconstructed image, which are:

1) *The Value of Gradient Descent Step*

We performed the algorithm with 60 view angle and parallel beam projections with different values of gradient descent step (alpha) as shown in Fig. (11), the resolution of the reconstructed images and Relative Error affected by the value of gradient descent step, the best value for this step is 0.01. Also we performed the algorithm with 60 view angle but using fan beam projections with different values of gradient descent step as shown in Fig.(12), the best value for this step with the lowest error is 0.1.

2) The Number of View Angles :

We test the algorithm with different numbers of view angles and with parallel beam projection for best value of gradient descent step =0.05 shown in Fig.(11), the result is shown in Fig.(13). we also test the algorithm for fan beam projections with best value of gradient descent step =0.1as shown in Fig.(12), the result is shown in Fig.(14), for both Relative error results, the lowest view angle (30 view angle) give the better results than ART, but as shown we could reconstruct images with very low error using 50 view angles.

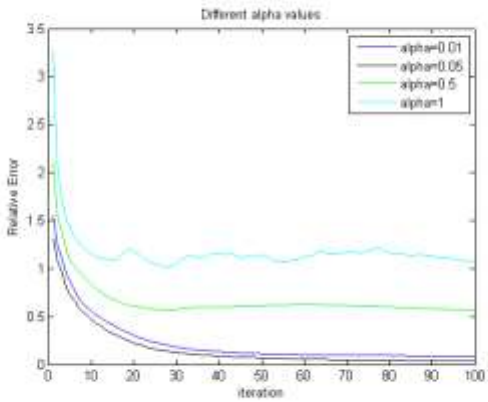

Figure 11: Relative Error for different values of gradient descent step “alpha” for parallel beam projections.

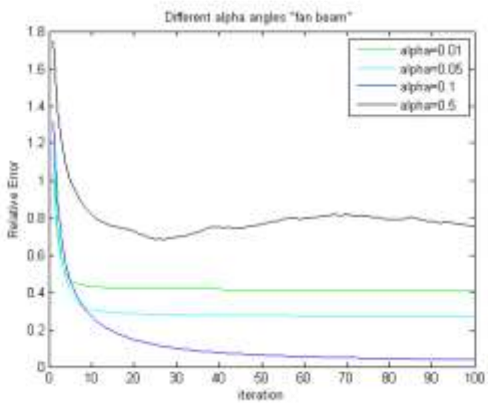

Figure 12: Relative Error for different values of gradient descent step for fan beam projections.

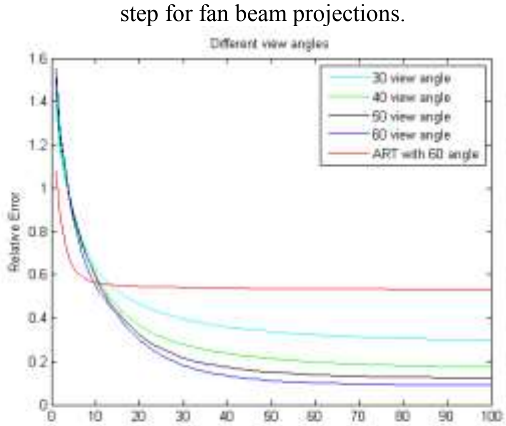

Figure 13: Relative Error for different view angles for parallel beam projections.

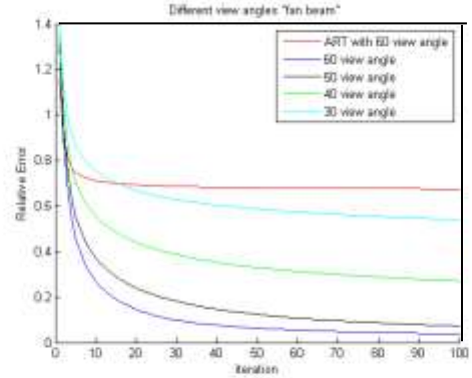

Figure 14: Relative Error for different view angles for fan beam projections

3) Sampling Patterns:

We test the effect of sampling pattern of view angles on the algorithm, we use random uniform distribution with parallel beam projection for best value of gradient descent step =0.05 and 60 view angle compared with normal random distribution for different values of variance as shown in Fig. (15), the best results with low error can be obtained using uniform random distribution, using any other distribution couldn’t reconstruct images with high resolution.

In addition to Relative Error and RMSE there exist other metrics to quantitatively assess the similarity between reconstructed images and the original phantom image. The universal quality index (UQI) [16], which is mathematically defined by modeling the image distortion relative to the reference image as a combination of three factors: loss of correlation, luminance distortion, and contrast distortion. The correlation coefficient (CC) [16], which is defined as a method that employs tracking techniques for accurate measurements of changes in images.

Table (1) shows the Quantitative quality metrics for traditional ART and CS iterative algorithm for parallel beam projections and 60 view angles.

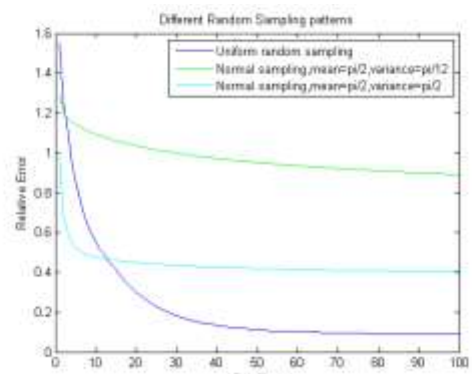

Figure 15: Relative Error for different parallel projections uniform random sampling and normal random sampling patterns.

Table 1: Quantitative quality metrics

	RMSE	**Relative Error**	**UQI**	**CC**
CS algorithm	0.0110	0.0350	0.9986	0.9975
ART algorithm	0.0646	0.5315	0.8516	0.9054

IV. DISCUSSION

The main aim of this paper is to reconstruct an image of digital Shepp-Logan phantom using small number of projections with high resolution compared to low dose traditional ART technique and to study the effect of controlling parameters on the reconstructed image. We could reach higher accuracy than others [17] by selecting optimal control parameters using parallel beam also we use CS algorithm to reconstruct images using fan beam orientations shown in the above results, measuring the relative error is more relevant than RMSE as it shows the noise with respect to the mean pixel value ,RMSE and quantitative measurements show best results for iterative CS proposed algorithm by applying ART update equation and L1 norm for sparse image.

As we mentioned before using any iterative algorithms as ART with few number of projections will lead to noisy images as shown in Fig.(7) and Fig.(9). To remove these artifacts we applied L1 norm and use gradient descent algorithm after ART algorithm is applied, the resolution of the reconstructed image depends on the gradient descent step, for large values of the gradient descent step the algorithm will be unable to converge and to find the solution of the required image and for small values the algorithm will take long time to converge so the optimal case to make the gradient step constant number multiplied by the error between the true and calculated projections as shown in Fig. (11) and Fig. (12), the results show that we could decrease error to low level about 0.01 while it is about 0.06 for traditional ART algorithms, the optimal gradient decent step is 0.05 for parallel beam and 0.1 for fan beam, using these values we could find the required image with high quality and low artifacts, using both orientations give different optimal values of alpha due to their different sinograms and different artifacts.

As the view angles increase the quality of image increases and relative error decreases as shown in Fig. (13) and Fig. (14) as the large number of projections represents the complete data set needed, we could reconstruct images with high quality and low relative error (about 0.01) from 40 view angles which is less than half the value of needed projections for traditional ART for both fan beam and parallel beam projections. Also using 30 view angles gives better results than traditional ART.

To reconstruct image using CS and to avoid aliasing we should use random distributions, for parallel beam we use uniform random sampling in view angles but for fan beam we use uniform random sampling for rotation angle, all the above results we use these sampling criteria. We test the algorithm for parallel beam and all optimal controlling parameters using different sampling patterns uniform and normal random distribution, normal random distribution wouldn't give high quality reconstructed image as the main idea of reconstruction is to get samples representing data not to be concentrated at specific regions as shown in Fig. (15). For normal random distribution as the variance decrease we get low quality reconstructed image as it doesn't represent the whole data and uniform random distribution gives the best results.

The RMSE is widely used for measuring reconstruction accuracy, whereas the UQI and CC can be used for evaluating the pixel-to-pixel similarity between reconstructed and original images. When assessing the image's quality, we demand the RMSE index to be as small as possible, while expecting the UQI and CC to have the contrary results.

Table (1) suggested that the CS algorithm with the best controlling parameters give reconstructed image more similar to the original image than the traditional ART algorithms with low RMSE and Relative Error and high UQI and CC.

The most critical parts in the implementation of CT CS iterative algorithm are the sampling criteria, projections formation using matrix/line geometry, the selection of the sparse domain and selection of iterative algorithm using to solve the sparse constrain. Applying the sparsity definition enables us to choose the most suitable sampling criteria which were the uniform random sampling as we mentioned before and by testing this sampling pattern for both view angles and parallel lines, we could reach best results with high resolution reconstructed image. For the selection of the iterative algorithm we decided to use gradient descent as it is very simple to implement, we tried to use the conjugate gradient as it could converge fast but we faced a problem for how to apply it to our equations so it could be part of the future work. Also using numerical solution for the derivative of gradient descent gives approximate solution so we tried to use the Fourier transform but we faced some problems in implementation also it could be part of the future work .

V. CONCLUSION

In Conclusion, We developed a CS iterative algorithm to reconstruct an image with view angles less than half of the needed projections used in all traditional algorithms; also we compare the effect of some controlling parameters on the quality of the reconstructed image.

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