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Compressed Sensing: Doppler Ultrasound Signal Recovery by Using Non-uniform Sampling & Random Sampling

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ABSTRACT

Several authors have shown that it is possible to reconstruct exactly a sparse signal from a fewer linear measurements, this method known as compressed sensing (CS). CS aim to reconstruct signals and images from significantly fewer measurements. With CS it's possible to make an accurate reconstruction from small number of samples (measurements). Doppler ultrasound is an important technique for non-invasively detecting and measuring the velocity of moving structure, and particularly blood, within the body. Doppler ultrasound signal has been reconstructed with CS by using random sampling and non-uniform sampling via $\ell 1$-norm to generate Doppler sonogram. The result show that the recovered signals with non-uniform sampling are the same as the original signal, there is a loss of very small peaks, when random sampling used for recovering the signals, there is no significant different between the original signal and reconstructed one when we used more than 85% of the data, when less than 85% of the data used, the reconstructed signals and the original signal are different. The sonograms generated from the reconstructed signals with random and non-uniform sampling are same as the original one, but there are some losses in contrast. The error of the reconstructed images was calculated, the result shows that the error in the image decreased with increasing the number of samples.

Keywords: compressed sensing; sparsity; Doppler ultrasound; $\ell 1$-norm; non-uniform sampling; signal & image processing; random sampling.

I. INTRODUCTION

Signal acquisition and reconstruction are a heart of signal processing, and sampling theorems provide the bridge between the continuous and the discrete-time worlds. The traditional approach of reconstructing signals or images from measured data follows the well-known Shannon sampling theorem [1, 2, 3] which states that the sampling rate must be twice the highest frequency. Similarly, the fundamental theorem of linear algebra suggests that the number of collected samples (measurements) of a discrete finite-dimensional signal should be at least as large as its length to ensure reconstruction. This principle underlies most devices of current technology, such as analog to digital conversion, medical imaging or audio and video electronics. The novel theory of compressed sensing (CS) also known under the terminology of compressive sensing, compressive sampling or sparse recovery provides a fundamentally new approach to data acquisition which overcomes this common wisdom. It predicts that certain signals or images can be recovered from what was previously believed to be highly incomplete measurements (information).

Compressed sensing is a new sampling theory that uses a fixed set of linear measurements together with a non-linear recovery process. In the last few years, an alternative theory of compressive sampling has emerged which show that the super-resolved signals and images can be reconstructed from far less data/measurements than what is usually considered necessary [4, 5]. To work with a low number of measurements, compressed sensing theory requires the sensed signal to be sparse in a given orthogonal basis and the sensing vectors to be incoherent with this basis. The theory of compressed sensing has been proposed by Candes and Tao [6, 7] and Donoho [8, 9]. From general viewpoint, sparsity and, more generally, compressibility has played and continues to play a fundamental role in many fields of science. Sparsity leads to efficient estimations, efficient compression and dimensionality reduction and efficient modeling [5, 10]. There are two main components of compressed sensing; the sampling strategy and the reconstruction algorithm [11]. Sampling involves measuring a quantity at regular intervals; the concept of sampling in compressed sensing is much more general. Sampling in compressed sensing consists of making random linear projection of the signal into a low dimensional space. The difference between conventional sampling and compressed sensing is that the reconstruction operator is nonlinear. Essentially these select the significant for some sparse representation and then calculate the least square approximation using the associated basis functions. Many theories of compressed sensing [12, 13, 14] have concentrated on proving that the near best performance is possible by using either a convex relaxation that boils down to solving a linear or quadratic program or greedy algorithms that interactively select the coefficients in a greedy way one at a time or in groups.

In this paper we used a Doppler (dopplersignal1.mat) spectrum data to reconstruct the Doppler spectrum by using CS instead of using the traditional approach of signal or image reconstruction. The signal will be reconstructed by using two different methods, by using random (uniform) sampling and non-uniform sampling methods, then the Doppler spectrum signal reconstructed by using a few numbers of measurements instead of using all numbers of measurements

II. PRINCIPLE OF DOPPLER ULTRASOUND

The basis of Doppler ultrasonography is the fact that reflected / scattered ultrasonic waves from a moving interface will undergo a frequency shift the magnitude and the direction of which will provide information regarding the motion of this interface. Doppler instruments generate either continuous wave (CW) or pulsed wave (PW) ultrasound. Other type of ultrasound wave such as shear or surface waves are rarely applied in medical ultrasonic's. Shear wave is strongly attenuated in soft tissue. From the point of view of Doppler techniques, the parameters that describe a wave, i.e., amplitude, frequency and phase, are important. Frequency and phase are more important for Doppler methods since the velocity of blood is obtained from the shifts in frequency and changes in phase of scattered wave [15]. The Doppler ultrasound is an important technique for non-invasively detecting and measuring the velocity of the moving structures, and particularly blood within the body, and is becoming indispensable tool in many diagnostic situations. The developments in Doppler technology have led to a vast increase in the number of non-invasive blood velocity investigations carried out in all area of medicine.

Nowadays there is a range of methods available for obtaining a pictorial record of a Doppler shift signal of, which is the best and most commonly used in real-time spectral analysis. The output of the spectral analyzer is usually represented as a sonograms, shown in figure 1. The horizontal axis represents time (*t*), the vertical axis frequency (*f*), and the intensity at co-ordinates (*t, f*). Both time-varying maximum frequency and mean frequency envelopes may also be extracted from the output of the analyzer. The maximum frequency envelope, or outline of the Doppler spectrum versus time, is the most commonly used parameter for Doppler waveform analysis, while the intensity-weighted mean frequency envelope is mostly common used for computing blood flow velocity and volumetric flow.

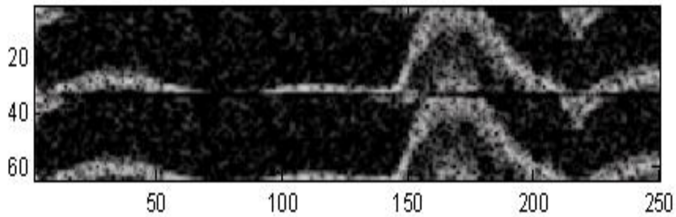


Fig. 1: Doppler Sonograms, created by using Doppler data

III. DOPPLER SPECTROGRAM

The Doppler shift frequency is proportional to velocity, and under ideal uniform sampling conditions the power in a particular frequency band of the Doppler spectrum is proportional to the volume of blood moving with velocities that produce frequencies in that band, and therefore the power Doppler spectrum should have the same shape as velocity distribution plot for the flow in the vessel. The variation in the shape of the Doppler power spectrum as a function of time is usually presented in the form of sonograms shown in figure 1 [15, 16].

Spectral Doppler ultrasound velocimetry involves systematic analysis of the spectrum of frequencies that constitute the Doppler signal. The Doppler frequency shift signal represents the summation of multiple Doppler frequency shifts backscattered by millions of red blood cells, which represent a bout 45% of the volume of blood. The Doppler signal is processed in sequential steps, consisting of reception and amplification, demodulation and determination of directionality of flow, and spectral processing [15, 17, 18]. The returning signals are first received and amplified by radiofrequency (RF) receiving device. The amplified signals contain of Doppler-shifted frequencies and carrier frequency, extracting carrier frequency from Doppler-shifted frequencies known as demodulation. There are various methods of demodulation [15, 17]. Qadrature sampling is needed to differentiate between flow toward the transducers (positive Doppler shift) and flow away from transducers (negative Doppler shift). The resulting signal consists of not only Doppler frequency shift, but also low-frequency/high-amplitude signal and high-frequency noise. Applying high-pass filter will eliminates the extrinsic low-frequency component of Doppler signals, and low-pass filter allows frequencies only below a certain threshold to pass, thereby removing any frequencies higher than that level.

The above steps generate demodulated and filtered Doppler frequency shift signals display as a complex, the variation of amplitude over time shown in figure 2. Spectral analysis coverts this waveform to an orderly array of constituent frequencies and corresponding amplitudes of the signal. The amplitude approximately represents the number of scatterers traveling at a given speed and is known as the power of the spectrum. A full spectral processing that provides comprehensive information on both the frequency and its average power content is called then power-spectrum analysis. Various approaches are used for spectral processing [19, 20]. There are numbers of factors that distort the power spectra and which may limit the accuracy with which the velocity distribution in a vessel can be determined. Under ideal sampling conditions the Doppler spectrum would have same shape as histogram of the velocity distribution of the erythrocytes within the Doppler sample volume [15].

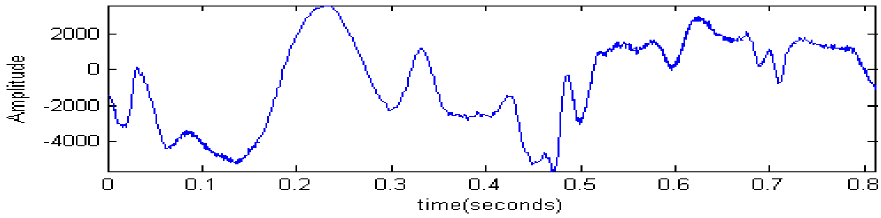


Fig. 2: demodulated Doppler signal prior to spectral processing.

IV. SPARSITY

Many natural signals have concise representation when expressed in a convenient basis. Roughly speaking one compresses the signal by simply keeping only the largest coefficients [21]. The problem of finding the sparsest representation or approximation in terms of the given dictionary turns out to be significantly harder than in the case of sparsity with respect to bases where the expansion coefficients are unique. Indeed, in [22, 23] it was shown that the general $\ell 0$-problem of finding the sparsest solution of undetermined system is 'NP-hard'. Greedy strategies such as Matching Pursuit algorithms [24] and $\ell 1$- minimization [8, 25, 26] were subsequently introduced as tractable alternatives. Sparsity is a fundamental modeling tool which permits coefficient fundamental signal processing [13].

V. COMPRESSED SENSING

Sparse signal can be approximately reconstructed efficiently from small number of non-adaptive linear measurements. This process is known as compressed / compressive sensing (CS). CS is an emerging method in computational signal processing. CS was first proposed in literature of information theory and approximation theory [8, 26]. In CS a few numbers of measurements of the signal samples will be considered to reconstruct the signal. This signal can be reconstructed with a good accuracy from these measurements by a non-linear procedure. Many authors [6-9, 27, 28, 29, 30, 31] have proposed the idea of acquiring signal in a compressive form. K-space signal and closely approximate compressible signals can be exactly recovered with high probability via the $\ell 1$ optimization [8, 26]. Convex optimization problem that conveniently reduces to a linear program known as basis pursuit [8, 12, 26] whose computational complexity is a bout O(N3).

VI. SIGNAL RECOVERY

Doppler (dopplersignal1) data from H. Torp experiment which has a length of 2032 were used to reconstruct Doppler spectrogram. MATLAB program was used to generate Doppler spectrum. All CS reconstructions were carried out in Matlab using the non-linear conjugate gradient methods. The Optimization based on $\ell 1$-norm was used to recover exactly the Doppler signal. Two different Matlab code (methods) were used. In the first method we used random (uniform) sampling to reconstruct the Doppler signal by using different size of data, between 80% - 5% of all data to get the recovered signal, which is used to generate Doppler spectrogram. In the second method non-uniform sampling was used to get recovered Doppler signal by using same size of data as in the first method to generate Doppler spectrogram. The file dopplersignal1.mat was loaded into Matlab. *iq* represent *Nx1* vector (Amplitudes), the transpose of *iq* is *x 1xN* vector and t is the time. The coefficient matrix A of size *KxN* generated from a random/non-uniform sparse model, the generated signal described as: $f = A\,x$, the noise z added to the signal.

For each CS sparse signal we obtain the measurements $f = A\,x$ and calculate estimated signal using $\ell 1$-norm. Different numbers of measurements (5%, 20%, 40%, 60%, 80%) were used to generate the recovered signal; above all these measurements all data (100%) were used, the recovered signals are shown in figures 3, 4, and 5. The reconstructed signals were used to generate Doppler spectrum's shown in figure 6 and 7, then the error from each reconstructed image was calculated, the results are shown in figures 8 and 9.

VII. Result & Discussion

The recovered signals results were shown in figures 3 - 5. The signal with length of 2032 and various numbers of measurements (128, 400, 813, 1219, 1625 and 2032, represent 5%, 20%, 40%, 60%, 80%, and 100%) were used to recover the signal. From figures 2 and 4, there is a small difference between the original signal and the reconstructed signals, the difference decreases with increasing the number of samples. When 100 % of data were used, the reconstructed signal is the same as the original signal; the reconstructed signal by using all data is shown in figure 3.

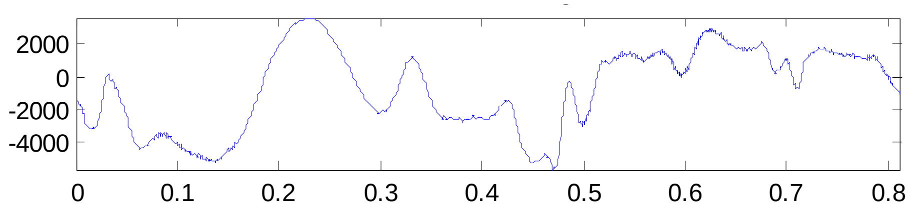
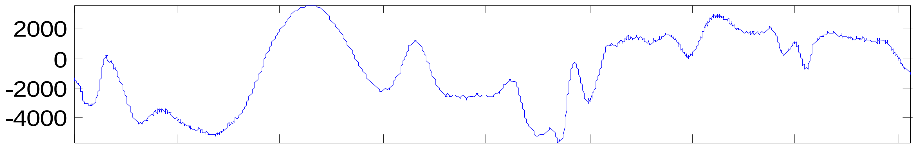
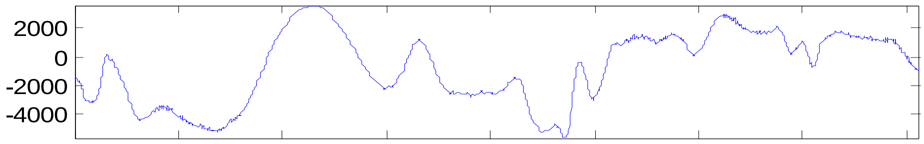


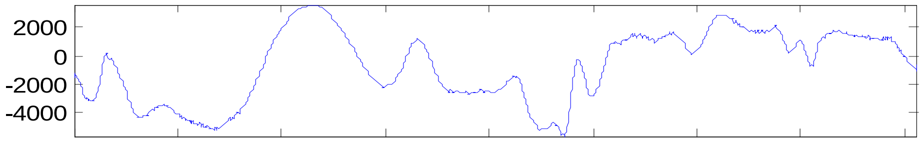
Fig. 3: Exact reconstructed signal with non-uniform sampling by using all data



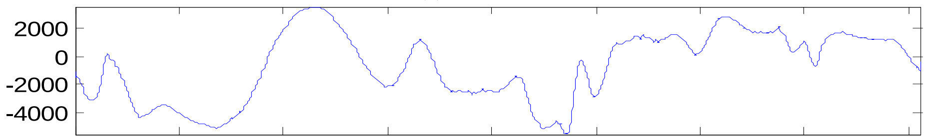
(a) 80 %



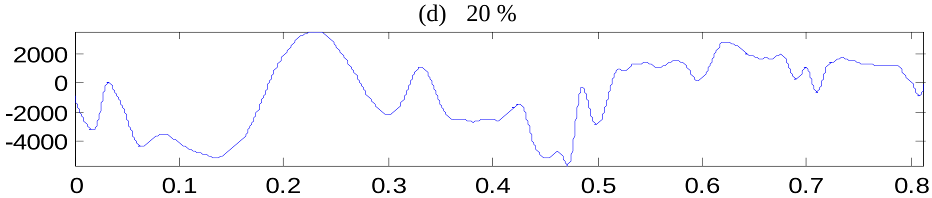
(b) 60 %



(c) 40 %



(d) 20 %



(e) 5 %

Fig. 4: Exact reconstructed signal with non-uniform sampling by using (a) 80 % of the data, (b) 60 % of the data, (c) 40 % of the data, (d) 20 % of the data, (e) 5 % of the data.

Figure 5 shows random sampling reconstructed signal, the result shows that there are big differences between the reconstructed signals and the original signal in figure 2. This variation decreased with increasing the number of samples. When all the data were used the resulting signal it is the same as the original signal.

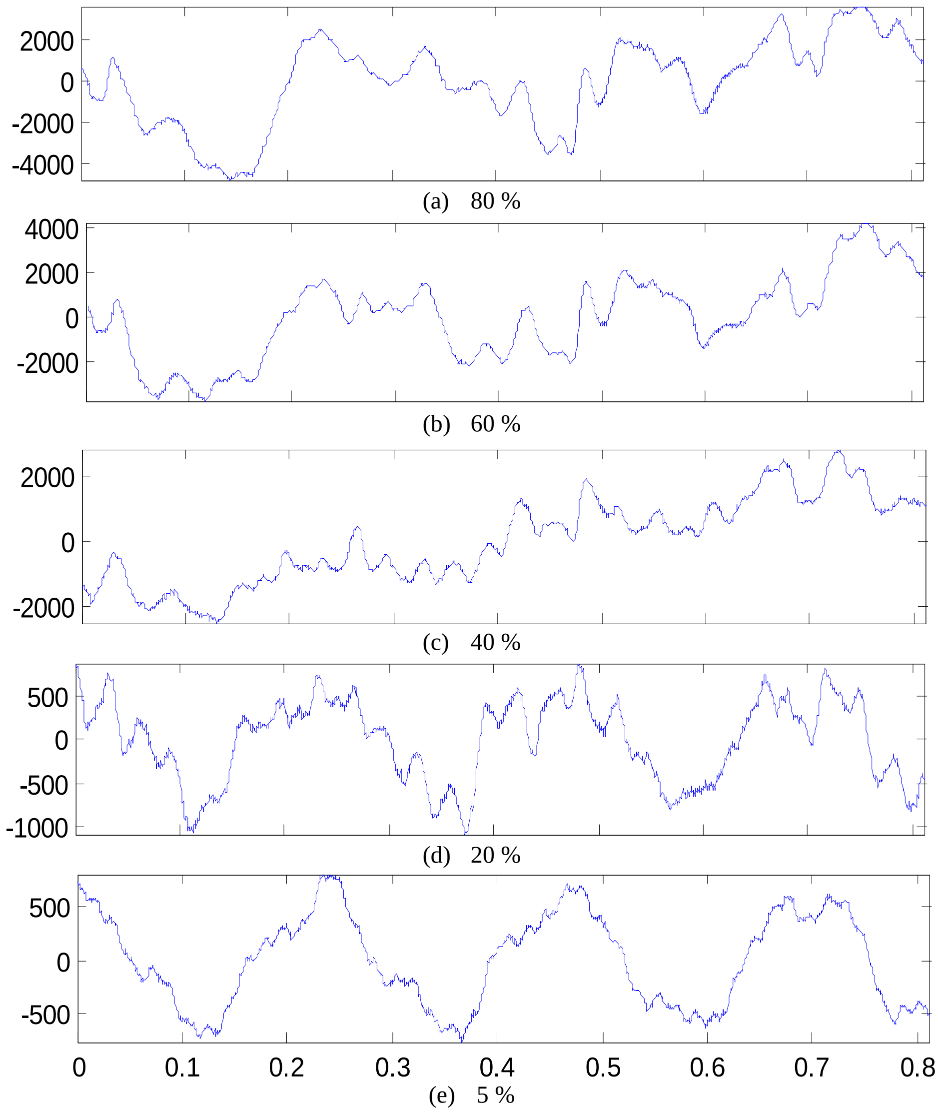
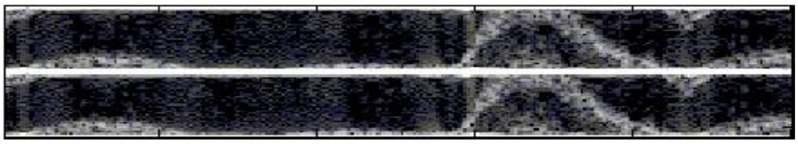
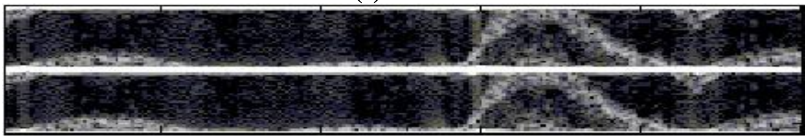


Fig. 5: Exact reconstructed signal with random sampling by using (a) 80 % of the data, (b) 60 % of the data, (c) 40 % of the data, (d) 20 % of the data, (e) 5 % of the data.

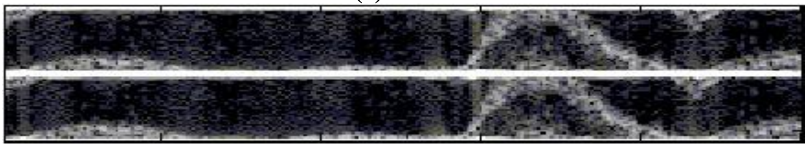
The original sonograms shown in figure 1 (shows the original Doppler sonograms); and the recovered sonograms signals were shown in figures 6 and 7 (by using non-uniform and random sampling). From the result we get exact recovery Doppler sonograms, but some loss of the low-contrast features in the uniform density. There is loss of image features for all the recovered signals (even by using all data for reconstruction).



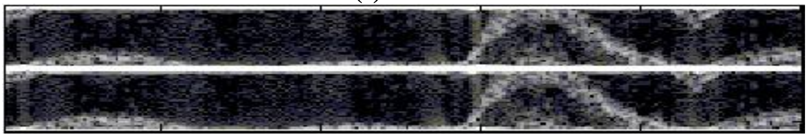
(a) 80 %



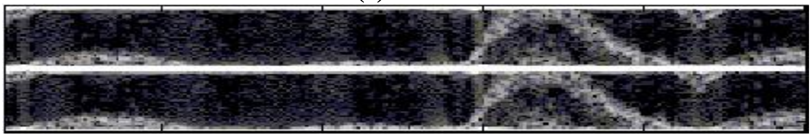
(b) 60 %



(c) 40 %

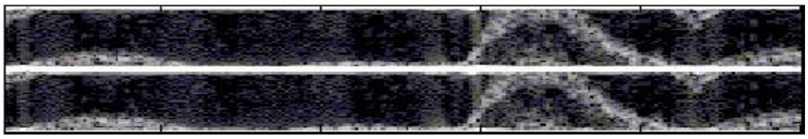


(d) 20 %

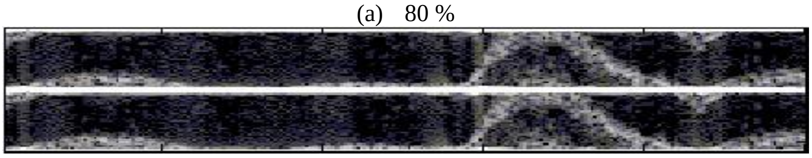


(e) 5 %

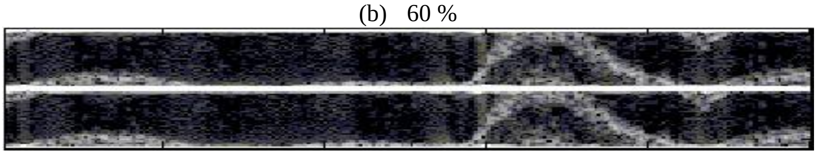
Fig. 6: Exact reconstructed sonograms images from recovered signals via $\ell 1$ norm with non-uniform sampling by using; (a) 80 % of the data, (b) 60 % of the data, (c) 40 % of the data, (d) 20 % of the data, (e) 5 % of the data.



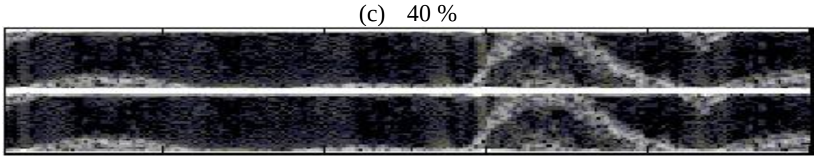
(a) 80 %



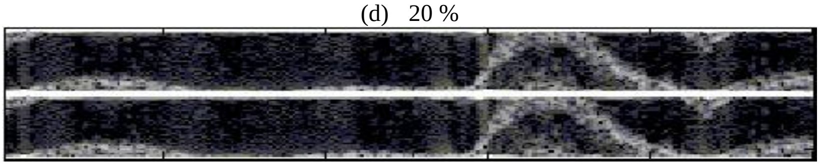
(b) 60 %



(c) 40 %



(d) 20 %



(e) 5 %

Fig. 7: Exact reconstructed sonograms images from recovered signals via $\ell 1$ norm with random sampling by using; (a) 80 % of the data, (b) 60 % of the data, (c) 40 % of the data (d) 20 % of the data, (e) 5 % of the data.

Figures 8 and 9 shows the error from the reconstructed images with non-uniform sampling and random sampling, respectively, the error is decreasing by increasing the number of samples; the reconstructed image with low number of samples has higher error. Comparing the error from non-uniform sampling and the error from random sampling, the error in the reconstructed sonograms by non-uniform sampling is less than the error of the reconstructed sonograms by random sampling.

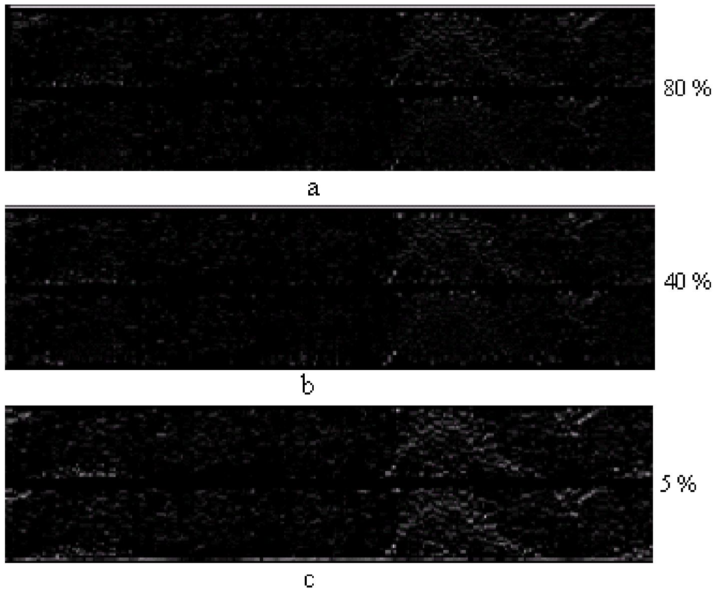


Fig. 8: Error of the reconstructed sonograms from non-uniform sampling by using; (a) 80 % of the data, (b) 40 % of the data, (c) 5 % of the data

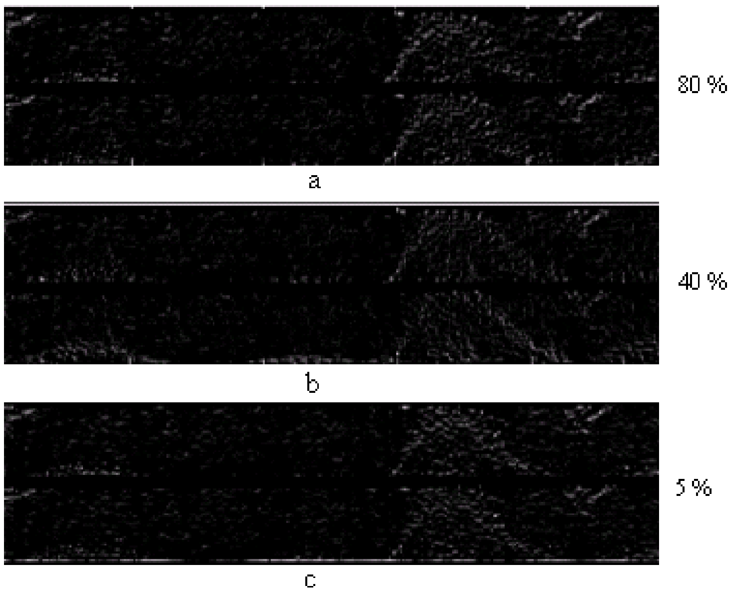


Fig. 9: Error of the reconstructed sonograms using random sampling by using; (a) 80 % of the data, (b) 40 % of the data, (c) 5 % of the data

VIII. CONCLUSION

The results show that the sampling is not the only way to acquire signals. When the signals of Doppler ultrasound spectrum are compressible or sparse it can be more efficient and streamlined to employ random (uniform) / non-uniform measurement and optimization to acquire only the measurement needed. The ability to reconstruct signals from very few measurements is important in signal processing. This paper describes the reconstruction of Doppler ultrasound signal by using CS measurement (random sampling and non-uniform sampling). With CS it's possible to use a very few number of measurements to reconstruct the signal in case of using a full data to reconstruct the signal. Applying CS to Doppler ultrasound spectrum is successful and gives a good result; the reconstructed sonograms are exactly same as the original (from random and non-uniform sampling). When the error of the reconstructed sonograms was calculated, the error is very low when non-uniform sampling used and decreased with increasing the number of samples used for reconstruction in both random and non-uniform sampling. Much greater performance and quality of spectrogram are expected by improving the Matlab code (used to recover the spectrogram signal) and using different methods of CS.

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