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OPTIMAL DESIGN OF RF PULSES WITH ARBITRARY PROFILES IN MAGNETIC RESONANCE IMAGING

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Abstract-**The proper design of RF pulses in magnetic resonance imaging has a direct impact on the quality of acquired images. Several techniques have been proposed to obtain the RF pulse envelope given the desired slice profile. Unfortunately, these techniques do not take into account the limitations of practical implementation such as limited amplitude resolution. Moreover, implementing constraints for special RF pulses on most techniques is not possible. In this work, we propose an approach for designing optimal RF under theoretically any constraints. The new technique poses the RF pulse design problem as a combinatorial optimization problem and uses efficient techniques from this area to solve this problem. In particular, an objective function is proposed as the norm of the difference between the desired profile and the one obtained from solving the Bloch equations for the current RF pulse design values. Two global optimization techniques were implemented using genetic algorithms and simulated annealing. The results show a significant improvement over conventional design techniques and suggest the practicality of using of the new technique for clinical use.**
Keywords - **MRI, RF pulse design, genetic algorithms, simulated annealing.**

I. INTRODUCTION

Magnetic resonance imaging (MRI) is one of the most promising imaging modalities. It offers true volumetric acquisition, ability to visualize and quantify flow, and spectroscopic imaging to image both anatomy and function. This technique relies on collecting the signal from an excited slice or volume within the human body. This excitation is achieved using special RF pulses that are designed to provide the required localization within the imaged volume. The proper design of RF pulses is important to avoid artifacts such as cross-talk in the acquired images.

The design of RF pulses is a rather difficult problem mathematically [1]. The basic goal of this design is to enable the desired slice profile to be achieved using the available RF pulse generation hardware. The practical implementation of RF pulse systems consists of a computer that stores the array of digital values representing the RF pulse envelope amplitudes. These values are converted using a digital-to-analog converter (DAC) into actual voltage levels. This DAC has a limited resolution in both amplitude and time. As a result, the generated envelope voltages appear like a piecewise constant curve with a fixed time step and limited stepwise amplitudes. These levels modulate the amplitude of the output of an RF generator before applying this output to the RF coils. The RF coils may be either linear (i.e., allowing only the real component to be applied) or quadrature (i.e., allowing both the real and imaginary components to be used). The actual frequency of the RF generator and the applied slice selection magnetic field gradient determine the position of excited slice. On the other hand, the amplitudes of the RF pulse envelope points determine the shape of the excitation profile as well as its flip angle.

For low flip angles, the problem of determining the RF pulse profiles can be approximated by the Fourier transform of the desired slice profile. An example of that is the use of Sinc-shaped RF pulse envelope to achieve a gate-like slice profile. As the flip angle gets higher, this approximation becomes inaccurate and more advanced pulse design techniques must be used [1,2]. Among the many design methodologies that have been proposed in the last decade, the Shinnar-Le Roux (SLR) method is probably the most widely used [2]. This method works by transforming the problem into one of designing a finite impulse response (FIR) filter. The solution of this problem is obtained as an FIR filter coefficients and subsequently transformed back into the desired RF pulse envelope.

Given the extensive literature on FIR filter design, the strength of the SLR technique is that it taps into some of the most powerful FIR filter design techniques to solve the original problem of RF pulse design [2]. Nevertheless, it still has some limitations that did not enable its performance to be optimal in practice. In particular, this technique does not take into consideration the limited amplitude resolution in designing the RF filter. As it is well known in FIR filter design literature, the limited precision (i.e., quantization or limited word length effects) in implementing the digital filter may substantially deteriorate its performance [3]. Even if the FIR filter design technique is modified to takes care of this problem and provide optimal filter coefficients for a given precision, there is no guarantee that the backward transformation to RF pulse coefficients would preserve this property for RF pulse coefficients. In other words, the characteristics of the SLR transformation do not allow such constraints to be imposed. In fact, it is generally difficult to impose any type of constraints on the solution (like for example adiabatic constraints). As a result, the obtained design may in fact be suboptimal in many cases that are common in practical use. An example where difficulty to obtain accurate slice profiles is reported is the use of unconventional spatial encoding techniques such as wavelet encoding and pseudo-Fourier imaging [4]. The implementation of such techniques had to compromise between the need to use low flip angles to obtain accurate slice profiles for correct encoding and the need for high flip angles for better signal-to-noise ratio. Therefore, a new RF pulse design technique that can incorporate practical constraints thus offering a true optimal performance under the practical implementation constraints would be rather helpful to solve these problems.

In this work, we formulate the problem of RF pulse design as a combinatorial optimization problem with an arbitrary number of constraints. This formulation takes into account the limited precision of RF pulse generation and provides the optimal results at any given precision. Unlike SLR technique, the objective function used is based on the computed slice profile, which offers a feedback loop to improve the results. Two optimization techniques are applied to solve this problem, namely, genetic algorithms and simulated annealing. We provide the detailed implementation details for each and present their results compared to those of the SLR technique.

II. THEORY

Given the definition of the RF pulse, it is possible to compute the expected slice profile using the solution to the Bloch equations. This solution relies on using the analytical form for the slice profile from a single rectangular pulse of arbitrary magnitude given in [1]. The Bloch equations relates the derivative of the magnetization M to the current magnetization value in the form,

$$\dot{\vec{M}} = \gamma \cdot \begin{bmatrix} 0 & -G_z & B_y \\ G_z & 0 & -B_x \\ -B_y & B_x & 0 \end{bmatrix} \cdot \vec{M} = \mathbf{A} \cdot \vec{M}. \qquad (1)$$

Here, G_z is the slice selection gradient, B_x and B_y are the two quadrature components of the RF pulse, γ is the gyromagnetic ratio. For a rectangular pulse, the solution can be simply computed as,

$$\vec{M}(z) = \vec{M}_0(z) \cdot \mathrm{Exp}(-\mathbf{A} \cdot t) \quad , \qquad (2)$$

where t is the duration of the rectangular RF pulse and the matrix exponent can be computed analytically for this problem as in [1].

Keeping in mind the practical implementation of RF pulses in the form of piecewise constant envelope pulses (i.e., a sequence of rectangular pulses of arbitrary amplitudes), the output magnetization from one piece serves as the initial condition for the next. Hence, given any design for the RF pulse, the slice profile can be computed this method. Given that the amplitudes of the RF pulses must be represented within a certain number of bits, the problem now becomes the one of finding the optimal combination of amplitudes that would give a slice profile closest to the desired. This problem description shows that this problem is indeed a combinatorial optimization problem. Using the rich literature of this area, the solution can be obtained efficiently and accurately. In this work, we explore two of the most prominent techniques in this area, namely, genetic algorithms and simulated annealing.

III. METHODS

A. Optimization Using Genetic Algorithms (GA)

The GA method employs mechanisms analogous to those involved in natural selection to conduct a search through a given parameter space for the global optimum of some objective function [5]. The main features of this approach are:

- A point in the search space is encoded as a chromosome.
- A population of N chromosomes/search points is maintained.
- New points are generated by probabilistically combining existing solutions.
- Optimal solutions are evolved by iteratively producing new generations of chromosomes using a selective breeding strategy based on relative values of the objective function for the different members of the population.

A solution is encoded as a string of genes to form a chromosome representing an individual. In many applications the gene values are [0,1] and the chromosomes are simply bit strings. An objective function, f, is supplied which can decode the chromosome and assign a fitness value to the individual the chromosome represents.

Given a population of chromosomes the genetic operators crossover and mutation can be applied in order to propagate variation within the population. Crossover takes two parent chromosomes, cuts them at some random gene/bit position and recombines the opposing sections to create two children. For example, for one-point crossover crossing the chromosomes 010-11010 and 100-00101 at randomly selected position 3-4 gives 010-00101 and 100-11010. Another example is for two-point crossover, where crossing the chromosomes 01-011-010 and 10-000-101 at randomly selected two positions 2-3 and 5-6 gives 01-000-010 and 10-011-101. Mutation is a background operator, which selects a gene at random on a given individual and mutates the value for that gene (for bit strings the bit is complemented).

The search for an optimal solution starts with a randomly generated population of chromosomes and an iterative procedure is used to conduct the search. For each iteration, a process of selection from the current generation of chromosomes is followed by applying the genetic operators and re-evaluation of the resulting chromosomes. Selection allocates a number of trials to each individual according to its relative fitness value. The fitter an individual the more trials it will be allocated and vice versa. Average individuals are allocated only one trial. Trials are conducted by applying the genetic operators (in particular crossover) to selected individuals, thus producing a new generation of chromosomes. The algorithm progresses by allocating, at each iteration, ever more trials to the high performance areas of the search space under the assumption that these areas are associated with short subsections of chromosomes which can be recombined using the random cut-and-mix of crossover to generate even better solutions [5].

The use of GA is robust in that they are not affected by spurious local optima in the solution space. Nevertheless, The parameters that control the GA can significantly affect its performance, and there is no guidance in theory as to how properly select them. The most important parameters are the population size, the crossover rate, the mutation probability and the fitness function. The following are the proposed algorithm details.

1) *Population of the chromosomes*: Population represents the size of the solutions that we are working with. The bigger is the population size, the faster the convergence, but the more the computational burden at the initial iterations. It can be fixed or dynamically changed allover the run of the algorithm. In our problem, a fixed population size of 1000 individual has been used.
2) *Population Initialization*: The chromosomes are initialized randomly. A chromosome that represents the SLR solution is added to the initial population.
3) *Chromosome structure*: Binary chromosome is used. The RF pulse is encoded into the chromosome as follows. The real RF pulse values are converted into discrete ones according to the resolution of the D/A of the MRI machine (12 or 16 bit for example). Every bit represents a gene. The most significant bits of all values are placed adjacent to each other, then the second most bits and so on until placing the least significant bits together at the end. Hence, the chromosome size equals the number of the RF pulse envelope values times the bit resolution (usually 12 or 16).
4) *Fitness criterion*: The chromosome is decode to obtain the corresponding RF pulse envelope amplitudes and its slice profile is computed by solving the Bloch equations. Then, an error measure is calculated for the difference between the response of this RF pulse and the desired response. This measure is usually taken as either the 1-norm or 2-norm of the difference vector. The results of this paper were obtained using the 1-norm.
5) *Selection scheme*: A biased roulette-wheel selection is used. In this method, the fitness of all the chromosomes in the population is evaluated. Divide each of these fitness values by the total summation of these fitness values and then multiply it by 100 to get the percentage that this chromosome can be evolved in the next generation.
6) *Crossover*: One point crossover is used the probability of crossover is taken as 90%.
7) *Mutation*: The probability of mutation is taken as small as 1%.

B. Optimization Using Simulated Annealing (SA)

The concepts of simulated annealing are based on a strong analogy between the physical annealing process of solids and the problem of solving large combinatorial optimization problems. Annealing is a thermal process for obtaining low energy states of a solid in a heat bath. It contains two steps. First, the temperature of the heat bath is increased to a maximum value at which the solid melts. Second, it is decrease carefully until the particles arrange themselves in the ground state of the solid. In the ground state the particles are arranged in a highly structured lattice and the energy of the system is minimal. The ground state of the solid is obtained only if the maximum temperature is sufficiently high and the cooling is done sufficiently slow. Otherwise the solid will be frozen into a meta-stable state rather than ground state.

The Metropolis algorithm is a numerical technique that simulates the annealing process [6]. In this algorithm, given a current state i of the solid with energy E_i, then a subsequent state j is generated by applying a perturbation mechanism which transforms the current state into a next state by a small distortion (for instance by displacement of a particle). The energy of the next state is E_j. If the energy difference is less than or equal to 0, the state j is accepted as the current state. If the energy difference is greater than 0, the state j is accepted with a certain probability which is given by $\text{Exp}((E_i-E_j)/(K_B.T))$, where T denotes the temperature of the heat bath and K_B is a physical constant known as the Boltzmann constant. The acceptance rule described above is known as the Metropolis criterion and the algorithm that goes with it is known as the Metropolis algorithm.

We can apply the Metropolis algorithm to generate a sequence of solutions of a combinatorial optimization problem. For this purpose, we assume an analogy between a physical many-particle systems and a combinatorial optimization problem. This is based on the equivalencies that solutions in the combinatorial optimization problem are equivalent to states of a physical system, and that the cost of a solution is equivalent to the energy of a state.

The control parameter here is equivalent to the temperature. That is, if the energy difference is greater than 0, the state j is accepted with a certain probability which is given by the Metropolis criterion where $K_B.T$ is replaced by a single control parameter c. Initially the control parameter contains a large value. This means that most changes even those with large deterioration to the objective function will be accepted. The value of c is gradually decreased with each step until it approaches a very small value near 0. Then no deterioration to the objective function will be possible. This feature allows the simulated annealing algorithm to escape from local minima [6].

In order to apply SA to our RF pulse problem, we start with the RF pulse designed by SLR Algorithm. A new solution is generated by adding a vector of random numbers to the vector containing the RF pulse values. The random vector should be with small values to generate a new solution with energy (cost value) with small distortion of the old solution. The cost function used is the same as the evaluation function used in the GA. The control parameter c is decreased linearly with each step. The constraints on the solution are applied to the new values in every step to make sure that the solution is acceptable (e.g., the constraints of limited amplitude resolution). The new values are accepted or rejected according to the outcomes from the objective function and the Metropolis criterion. The process in repeated until the changes in the solution become very small.

VI. Results and Discussion

The proposed techniques were applied to design two RF pulses with rectangular spatial profiles at $\pi/2$ and π flip angles. The outcome of the SLR technique is compared to the outcomes from optimized RF pulses using the GA and SA methods. The results are shown in Figs.1 and 2. The 1-norm of the error between the desired and obtained is shown in Fig. 3. As can be observed, both techniques show a significant improvement over the design computed using the SLR technique. Also, the design using the SA method appears better than that from the GA technique. Even though both techniques should theoretically reach the global optimum, we have to realize that both techniques are iterative. This means that the user decides when the iteration stops both in time and accuracy. Hence, the above results from comparing GA and SA techniques indicate that the SA technique reaches the solution faster than GA in the number of experiments we performed.

One of the most important applications for the proposed technique is in the area of unconventional spatial encoding where complex RF pulse are to be generate at high accuracy. The proposed technique is expected to enable better reconstruction accuracy, less image artifacts, and higher signal-to-noise ratio. The study of this area requires the investigation of several of these techniques and the assessment of the results and is left to future work.

The addition of constraints on the solution using the proposed approaches is straightforward. In each of the algorithms for GA or SA, the obtained new solution after perturbation is passed through a 'filter' that checks the validity of the new solution from the point of view of the required constraints. In very much the same way this was used for practical amplitude resolution, other conditions can also be included for specialized RF pulse such as adiabatic pulses. The investigation of this application should be addressed in future work.

V. CONCLUSIONS

A new optimized RF pulse design approach is proposed. The new technique relies on posing the problem as a combinatorial optimization problem and uses genetic algorithms and simulated annealing to compute the solution under any type of constraints. The results demonstrate the success of the new approach and suggest its potential for practical use in clinical magnetic resonance imaging.

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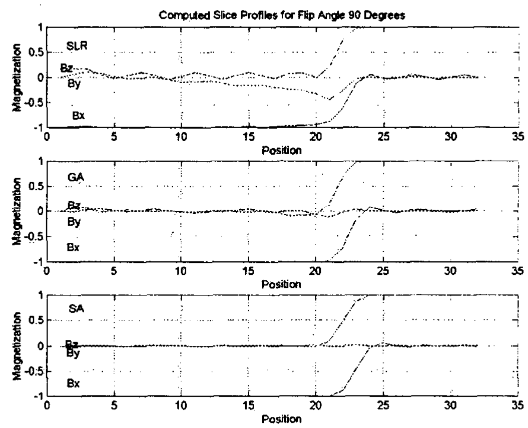

Figure 1: Comparison between slice profiles for flip angle of 90 degrees.

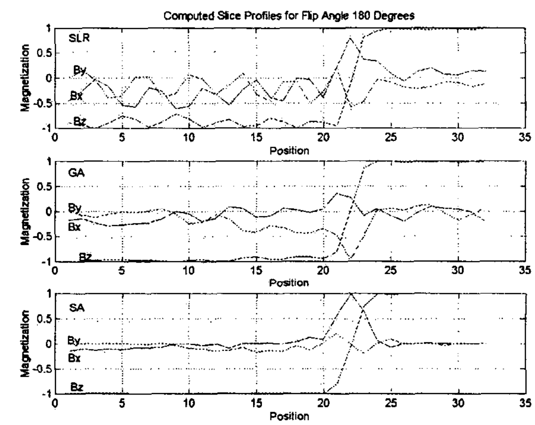

Figure 2: Comparison between RF pulses for flip angle of 180 degrees.

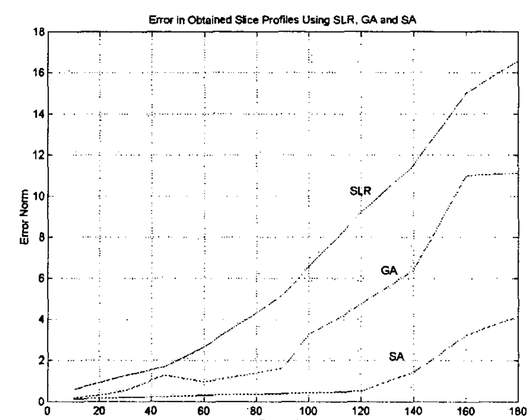

Figure 3: Comparison between error in slice profiles versus flip angle.