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Waveform and Beamform Design for Doppler Ultrasound Vector Flow Mapping *

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Abstract – **We consider the problem of producing an "optimal" range- 2-D Doppler flow map in a finite amount of time. A range- 2-D Doppler provides estimates of the magnitudes of the axial and two transverse flows in general as a function of range. By optimal image, we mean the closest image in a 2-norm sense to the actual the transverse and line-of-sight flows distribution as a function of range. We show that to obtain that optimal image we need to adjust both the beamform and waveform of the transmitted ultrasound field to the flow distribution. We propose a beamform and waveform selection strategy that leads to a progressive refinement of resolution of the produced image along the range, and vector velocity axes. We discuss imaging scenarios in which the returns due to the optimal beamforms and waveforms can be "synthesized" from those corresponding to a standard beamform and waveform and scenarios where one actually needs to generate different beamforms and waveforms to improve the quality of the final image.**

I. INTRODUCTION

Many authors have investigated the process of velocity vector estimation. Some appraoches required 3 transducers using frequency shift measurements, e.g., [1]. Others suggest to use an additional bandwidth measurement and using only two transducers [2]. The common factors in all vector velocity estimators is the need to have apertures that are not coplanar and the need to align them in such a way that their sample volumes intersect. This is obviously of considerable amount of difficulty to use in a practical medical imaging setup.

In [5], a model of the imaging process was developed and the possible velocity detection strategies were derived. One of the theoretical techniques discussed was the narrowband excitation/ narrowband field technique in which the velocity components can be estimated using Doppler shift measurements from at least an equal number of independent apertures. For example, if we need to estimate two components of the velocity, we have to use at least two apertures and so on. Using the general model of Duplex imaging introduced in [4], we shall develop the model for this special case and show that it gives a form that can be related to the classical range-Doppler ambiguity function. Hence, the principles of resolution refinement developed for radar imaging can be directly generalized to the process of Duplex imaging. Since adjusting the system parameters to an unknown map is difficult, we propose an alternative approach. In this approach, a progressively finer image of the Doppler map is obtained by transmitting a fixed set of different waveforms from independent apertures.

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II. THEORY

In [5], the process of duplex imaging was defined in terms of a generalized ambiguity function $\phi(.,.,.,.)$ which takes the following form without normalization:

$$\phi(x,z,v_x,v_z) = \int_{-\infty}^{\infty} \left[u(x^a + v_x^a t) \cdot u^*(x + v_x t) \right] \cdot \left[s\left(t\left(1 + \frac{2v_z^a}{c}\right) - \frac{2z^a}{c}\right) \; s^*\left(t\left(1 + \frac{2v_z}{c}\right) - \frac{2z}{c}\right) \right] dt \qquad (1)$$

Here u(.) is the ultrasound field at the depth of interest and in the direction of the projection of the motion path onto the image plane, s(.) is the temporal excitation, (x,z) are the coordinates of the moving particle in the image plane, v_x is the velocity component in the x-direction, v_z is the velocity direction in the z-direction. Assuming narrowband functions are used for u(.) and s(.), the following approximations are considered:

$$s\left(t\left(1 + 2\frac{v_z}{c}\right) - 2\frac{z}{c}\right) \approx s'(t - \tau_z) \cdot e^{j(\omega_c + \omega_z)t} \qquad (2)$$

$$u(x + v_x t) \approx u'(t + \tau_x) \cdot e^{j\omega_x t} \qquad (3)$$

where $\omega_z = 2\frac{v_z}{c}\omega_c$, ω_c = carrier frequency, $\tau_z = 2\frac{z}{c}$, $\tau_x = \frac{x}{v_x}$, and $\omega_x = v_x \cdot \nu_x$ is related to the spatial frequency of the beam pattern at the given range. Substituting from (2) and (3) into (1), we obtain:

$$\phi = \int_{-\infty}^{\infty} u'(t + \tau_x^a)\, u'^*(t + \tau_x) \cdot s'(t - \tau_z^a)\, s'^*(t - \tau_z) \cdot e^{j(\Delta\omega_x + \Delta\omega_z)t} dt \qquad (4)$$

where $\Delta\omega_x = \omega_x^a - \omega_x$ and $\Delta\omega_z = \omega_z^a - \omega_z$. Hence, if we define $\eta = \Delta\omega_x + \Delta\omega_z$. The above expression can be rewritten as:

$$\phi = \mathbf{F}\left\{ u'(t + \tau_x^a)\, u'^*(t + \tau_x) \cdot s'(t - \tau_z^a)\, s'^*(t - \tau_z) \right\}_{\omega=\eta} \qquad (5)$$

To observe the characterisitics of the above expression, we can write the Fourier transform expression as a convolution of two independent functions as:

$$\phi = \mathbf{F}\left\{ u'(t + \tau_x^a)\, u'^*(t + \tau_x) \right\}_{\omega=\eta} * \mathbf{F}\left\{ s'(t - \tau_z^a)\, s'^*(t - \tau_z) \right\}_{\omega=\eta} \qquad (6)$$

This expression is simply the convolution of the ambiguity functions of both the temporal and spatial signals s(.) and u(.). This means that the results from radar imaging can be readily applicable to do the improvement in both of these functions by using orthogonal set of waveforms. Notice also that the frequency shifts due to range and azimuth velocities are lumped together in the measurable η. Hence, we can see that the shape of the projection of the ambiguity surface onto the $\Delta\omega_x$-$\Delta\omega_z$ plane will be centered around the line $\Delta\omega_x = -\Delta\omega_z$ at which the two ambiguities peak together. Since both $\Delta\omega_x$ and $\Delta\omega_z$ are directly proportional to the excitation frequency ω_c with different proportionality constants, it can be shown that all the equations obtained from different excitation signals are linearly dependent and therefore it will not be possible to resolve $\Delta\omega_x$ and $\Delta\omega_z$ using this method. On the other hand, $\Delta\omega_x$ is uniquely dependent on other variables which are in general related to the aperture geometry and the range. Hence, by changing the aperture geometry and using another properly selected excitation, we can obtain progressively improved estimates of both velocity components.

III. IMPLEMENTATION & RANGE INVARIANCE

Assuming an exact Fourier transform relationship exists between the source aperture and the field at a certain depth ζ, a narrowband field intensity in a given direction can be obtained by using two identical coplanar transducers placed along the direction of interest in the source aperture. The distance between the two transducers directly determines the spatial frequency modulation at the focal plane. A direct generalization of this concept is using four tranducers, two along each of the x and y direction, to obtain a two-dimensional spatial frequency modulation.

To answer the question of range-invariance, consider the one-dimensional modulation case WLOG. The angular spectrum at the depth plane ζ is by definition an image of the source aperture from the exact Fourier transform relationship. That is, the spectrum is bandpass and real. The distance between the center of the bandpass spectrum and the origin defines the modulation frequency at this particular plane. From Fresnel propagation assumptions, the propagation from the focal plane to any other plane can be represented by a 2-D linear filter. As a result, the angular spectrum at any z will necessarily have the same modulation amount as the one at the focal plane, with the modulated spatial pattern being different. Consequently, the power spectrum obtained from this method is range-invariant.

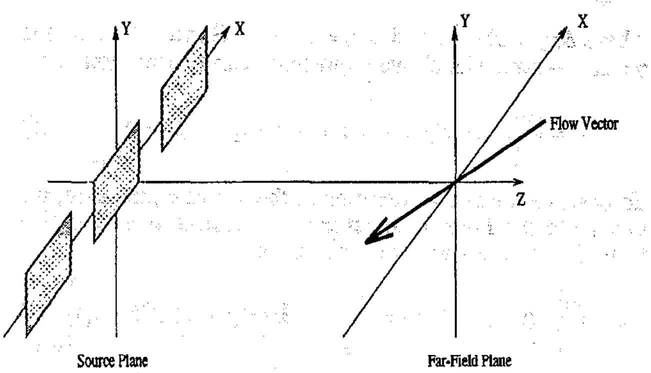

Fig.(1) Imaging Configuration

IV. SIMULATION RESULTS

Using the a linear array configuration, the one-dimensional version of our theorem can be investigated. If we consider the field of a three-element array in the far-field and use an absolute-value block right after the transducer in the reception mode, we can see that the spatial modulation obtained is proportional to the length of the transducer array. It should be noted also that this modulation is not range-invariant since the range-invariance condition is not met anywhere in the field. If we use gaussian apodization for each of these transducers, the resulting ambiguity function projection onto the $\Delta\omega_x$-$\Delta\omega_z$ plane takes the form in Fig.(1) for different numbers of transducer elements. As we can see, the minimum number of coplanar apertures that can be used to estimate the two separate components of the flow is two. Also, it can be seen that the use of more than two apertures is not a redundancy since it significantly improves the estimation process.

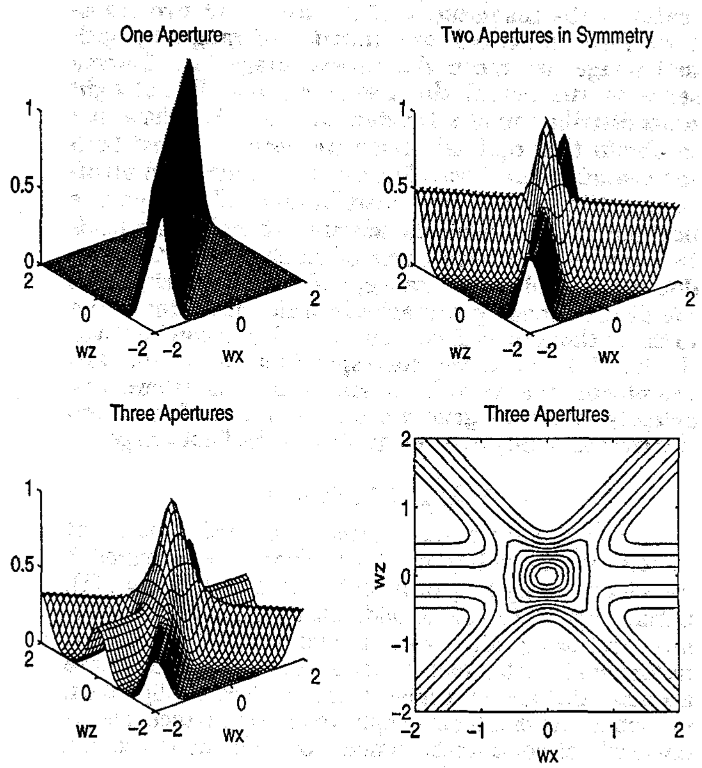

Fig.(2) Results of progressive improvement in velocity estimates

V. CONCLUSIONS

We have demostrated the progressive refinement of range and velocity vector components resolution can be obtained by using an orthogonal set of excitation signals and a set of independent apertures. The technique is readily applicable to one- or two-dimensional transducer arrays.

VI. REFERENCES

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