Space-Invariant True-Velocity Flow Mapping Using Coplanar Observations *

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Abstract – **The diagnostic value of the current two-dimensional Doppler flow mapping techniques is limited by the fact that velocity values at different parts of the image are only projections of the total three-dimensional true-velocity vectors onto the image plane. Since these vectors have different orientations, the resulting values do not have the same reference and hence cannot be compared for correct diagnosis. In this work, we show that it is possible to obtain true-velocity flow maps from coplanar observations. This effectively eliminates any angle dependency in the imaging process and provides a convenient way of representing velocity as a two-dimensional map relative to the same reference. Also, we establish the conditions under which this estimation process is space invariant to illustrate the practicality of the technique.**

I. INTRODUCTION

Many authors have investigated the effect of transit time and geometric broadening on the Doppler spectrum [1]. In the generalized model, these two effects are related and their contribution is essentially a function to the characteristics of the ultrasound beam and the path of the moving particles in the field. According to this model, the resultant Doppler spectrum depends on both the axial and the lateral components of the flow. Using the generalized Doppler model, the theory of estimating the transverse component of a Doppler flow was developed for ultrasound fields in the Fraunhofer zone of the transducer and in the focal plane of focussed transducers [2]. This theory suggests that the lateral component of the flow can be obtained by an equation very similar to that for the axial flow by using the absolute bandwidth of the return signal instead of the Doppler shift. Existing theoretical literature assumed a single particle moving transversely in the field and obtained the return spectrum by different methods such as diffraction theory and geometrical transducer argument. Some intuitive expressions have been given for the case of oblique flow, but no rigorous formulas were derived for this case.

Four problems arose during the above experiments suggesting that the conditions needed to obtain accurate transverse flow spectra are too strict and therefore of limited practical value. First, the flow path has to be exactly at the focal plane of the focussed transducer or at the far zone of unfocussed transducer. This means that we are unable to scan the whole image plane in the first case, or that the absolute bandwidth of the returned Doppler signal might be too small for accurate measurement. Second, the absolute Doppler bandwidth dependence on the range position has not been resolved. Some authors suggested that the absolute Doppler bandwidth is range invariant for the focussed transducer arrangement and showed some experimental results to prove that [3]. Third, the lateral position dependence of the flow path was not considered theoretically. Fourth, the effect of the temporal characteristics of the aperture was not investigated. These issues must be resolved to establish this technique.

II. GENERALIZED MODEL

Assume a line of random scatterers with a Poisson impulses distribution to be moving in a general direction in the imaging plane making an angle θ with the axial (z) direction. Choose the lateral direction x as the direction of the projection of the flow line on the transverse plane perpendicular to the z-axis at the depth of interest. Assume that the scatterers are identical perfect Rayleigh scatterers with scattering cross section σ_s and that they maintain a uniform velocity v throughout the effective period of insonification. Let the field magnitude along the path of scatterers be denoted as u(x, z), the temporal insonification window be s(t), and the random scattering line process be denoted as f(r). Then, the distribution of the scatterers across the flow line as a function of the distance to the observation point $r=\sqrt{x^2+z^2}$ can be given as:

$$f(r;x,z)=\sigma_s\sum_{n=-\infty}^{\infty}\delta(r-r_n)=\sigma_s\sum_{n=-\infty}^{\infty}\delta(x-r_n\sin\theta,z-r_n\cos\theta) \quad (1)$$

If we consider only small variations of the depth z around a given bias value z_o the dependence of the lateral beam profile can be considered as an exculsive function of x in this domain. That is, $u(x,z)\approx u(x,z_o)$ in the region of interest (We shall call $u(x,z_o)$ as u(x) for short). Then, the reflected signal from the process of insonifying the above process while moving at a uniform velocity $v=(v_x,v_z)$ can be expressed as:

$$s_r(t)=\int\int_{-\infty}^{\infty}u(x)\cdot f(x-v_xt,z-z_o+v_zt)\cdot s\left(t-\frac{z-z_o}{c}\right)dz\,dx \quad (2)$$

Hence, if the magnitude of the velocity vector is much smaller than the phase velocity of ultrasound in the medium, the received signal can be given in the form:

$$r(t)\approx\int_{-\infty}^{\infty}f(r)\cdot u(r\sin\theta+v_xt)\cdot s\left(t+2\frac{tv_z}{c}-2\frac{r\cos\theta}{c}\right)dr \quad (3)$$

Define the functions u′(.) and s′(.) as:

$$u'(\alpha)=u(-\sin\theta(\alpha+a-b)) \quad (4)$$

$$s'(\alpha)=s\left(\alpha\cdot\frac{2\cos\theta}{c}\right) \quad (5)$$

where: $a=-v_xt/\sin\theta$ and $b=ct(1+2v_z/c)/(2\cos\theta)$. Then, r(t) can be expressed as:

$$r(t)=\int_{-\infty}^{\infty}f(r)\cdot u'(b-r)\cdot s'(b-r)\,dr=f(b)*\left[u'(b)\cdot s'(b)\right] \quad (6)$$

where f(b) is a Poisson process with parameter $\lambda'=\lambda c(1+2v_z/c)/(2\cos\theta)$. Notice that the form in (6) models the process as linear filtering of a Poisson process, with filter impulse response defined by a multiplication of dilated/compressed versions of the temporal and spatial

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patterns of the insonification field. Hence, the average power spectrum of the returned signal can be expressed as:

$$\Phi_{RR}(\omega) = \frac{\lambda' \sigma_s^2}{\left(v_x \cdot (1 + 2\frac{v_z}{c})\right)^2} \cdot \left| U\left(\frac{\omega}{v_x}\right) * S\left(\frac{\omega}{1+2\frac{v_z}{c}}\right) \right|^2 \quad (7)$$

Here $\Phi_{RR}(\omega)$ is the Doppler power spectrum, λ' is the Poisson model parameter which can be related to the Hematocrit value (ratio of the volume of the formed elements to the total volume of Blood), σ_s is the scattering cross section of the individual scatterers, v_x and v_z are the transverse and axial components of the flow, U(.) is the Fourier Transform of the effective transmit-receive aperture at the depth of interest, S(.) is the Fourier transform of the excitation signal, c is the ultrasound velocity in the medium. Notice that we have assumed here that $S(\omega)$ is a bandpass signal and that the bandwidth of U to be small enough to make the convolution signal a bandpass signal to eliminate a dc term.

Some special cases of interest can be easily derived from the above formula. For example, the classical Doppler formula can be obtained from the above form by using a constant for u(.) and assuming s(.) to be a narrowband signal. Also, the formula obtained in [2] for the transverse velocity estimation can be obtained by assuming a narrowband s(.) and a zero axial velocity. More importantly, this formula suggests that the use of wideband functions for both s(.) and u(.) should be looked at more closely. As can be seen from the formula, the result of a steady flow in any given direction will be a bandwidth broadening which is directly proportional to the magnitude of the flow. This broadening is not only a function of the transverse velocity from the spatial beam pattern effect alone, but also of the axial velocity from the temporal excitation. Hence, we can directly measure the bandwidth for two (or more) different coplanar apertures and solve algebraically for both flow components. This can be done by using different rings of an annular array for example. Table.I lists the different possibilities of doing velocity magnitude measurements.

Table. I Theoretical velocity estimation methods

s(.)/u(.)	WB/WB	NB/WB	NB/NB
Measurement	BW	BW & FS	FS
Min. Req. Apertures	2	1	2

Here WB denotes wideband, NB denotes narrowband, BW denotes bandwidth and FS denotes frequency shift. Notice that for circularly symmetric apertures, the magnitude of the full-length velocity vector can be obtained in theory by estimating only its two components v_z and v_x from the return signal.

III. Space-Invariance Conditions

A potential problem with the above suggested methods is that the spatial beam pattern is a function of range in general. Also, the derivation assumed that the line of scatterers pass exactly through the origin of the range plane of interest. Any range or azimuth shifts could in general lead to a completely different spatial beam pattern u(.) in (7) and hence to different estimation process. This, if true, can greatly hinder the practical use of the technique. In the following, we shall derive the conditions under which the measurements are range and azimuth shift-invariant and hence the process can be of practical value.

Given that the wave propagation under Fresnel approximation can be represented by a linear system with a range-dependent space transfer function of infinite support [4], it can be shown that an aperture with finite-support angular spectrum can only propagate to another with the same support. Hence the absolute Doppler bandwidth remain finite and the same. On the other hand, if the field does not have this property, the absolute bandwidth is infinite and any other definition of the bandwidth is clearly range-dependent following the space transfer function. Hence, under Fresnel propagation conditions, the absolute bandwidth of the returned Doppler spectrum is range-invariant if and only if there exist a depth ζ at which the angular spectrum of the ultrasound field perpendicular to the direction of wave propagation has a compact support. We shall call the beams which satisfy this condition compact angular spectrum beams (CASBs). Note that for any CASB, the Fourier transform of any laterally-shifted slice of the field at any depth is the projection of the the compact angular spectrum at that depth multiplied by a linear phase term onto the same direction defined by the angle of the slice. This means that we obtain exactly the same absolute bandwidth as the original function only for circularly symmetric apertures since asymmetrical functions give different projections at different angles.

Note also that the above condition can be met in a variety of ways. For example, CASB can be produced by placing the transducer at a focal length distance in front of a thin lens [4]. Also, they can be produced by using annular arrays or two-dimensional arrays by proper choice of their excitation. It should be noted that the above conditions are different from the result in [3] which was based on the conventional transducer-against-lens configuration or far field. Moreover, we can show that the far field of an unfocussed transducer can be shown to be range-varying [4]. These situations have extremely rapid oscillating quadratic phase terms in their spatial expressions, which translate to infinite angular spectrum that has a strong range-dependence.

IV. Conclusions

We have derived a closed form expression for the power spectrum of the return signal from a generalized flow direction, temporal excitation and spatial field pattern. Based on this model, the different strategies of velocity estimation were derived. We have also established the conditions under which the estimation process is space-invariant, which can be readily met using the existing transducer arrangements. This shows the value and practicality of those techniques for direct clinical use under these conditions.

V. References

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