A Local-Field Extrapolation Algorithm for Improving the Spatial Resolution in Magnetic Resonance Dynamic Imaging

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ABSTRACT

In magnetic resonance imaging (MRI), data is collected as spectrum samples. The acquisition time is proportional to the number of the spectrum lines. Therefore, only few lines of the data space may be required in order to track rapid changes of an object. In the current techniques, the missed lines may be zeroed or replaced by the corresponding lines in a reference image, which is acquired *a priori* for the same anatomical cross-section. However, this always comes at the expense of the spatial-resolution. In this study, we propose an extrapolation iterative algorithm to provide an improved estimate of the missed lines. Additional spatial and spatial-frequency constraints of the reference image are incorporated to enhance the convergence and obtain a better estimate of the initial conditions of the iterations. Results from simulated data verify the theory and indicate that the algorithm may provide better reconstruction in dynamic imaging studies.

Key words: extrapolation, magnetic resonance, dynamic imaging.

1. INTRODUCTION

Several MRI applications involve imaging a dynamic object to reveal its temporal behavior. Examples include monitoring tumor uptake of a contrast agent, functional MRI, and cardiac imaging. In such applications, it is often difficult to achieve the desired temporal resolution while maintaining the desired spatial resolution and field-of-view (FOV). Most of the dynamic imaging techniques begin with acquiring a full spatial resolution image, called reference image, then continue to acquire dynamic images at high temporal resolutions. The basic idea of reducing the acquisition time in these techniques is to assume the presence of some common information between the reference image and the subsequent dynamic images. Therefore, information from the reference image can be partially used to reconstruct the subsequent dynamic images thereby achieving a considerable saving in the acquisition time.

Keyhole and reduced FOV (rFOV) techniques are commonly used in MRI dynamic imaging. Both techniques follow the main procedures discussed above and operate in the spectral domain of the image, namely the k-space. The Keyhole technique [1] updates only the central k-space part of the acquired dynamic images, assuming that the outer regions are stationary. Alternatively, the reduced FOV technique [2] assumes that the dynamic changes occur only in the central region of the image itself. This assumption is used to improve the temporal resolution of the central region of a dynamic image while maintaining the same spatial resolution throughout. Much work was inspired by these two techniques in order to improve the image quality. However, because the stationarity assumption is usually unrealistic, the reconstructed images suffer from degradation in the spatial resolution as observed in keyhole [3], and aliasing as encountered in the rFOV technique [2].

In this work, we propose an iterative algorithm that takes advantage of both the keyhole and the rFOV techniques. Towards this end, an extrapolation algorithm is used to mitigate the possible discontinuity resulting from concatenating parts of two images, either in the k-space or the image-space. Extrapolation of missing spectral components of a finite extent object has been of particular interest since the 1960s. The Gerchberg-Papoulis (GP) algorithm is a classical example of such algorithms [4,5].

A generic block diagram of the GP algorithm is shown in figure 1. The algorithm starts by obtaining the inverse Fourier transform, i.e. the image, of the measured spectrum, which represents part of the complete spectrum. The resulting image is then truncated because the imaged object is originally of finite extent. The truncation process can be viewed as a constraint on the image. Next, the Fourier transform is obtained for the constrained image and the known segment of the spectrum replaces its counterpart in the last iteration. The process is repeated until satisfactory results are obtained. This algorithm has the drawback of being slow to converge to the true image. Another disadvantage is its sensitivity to additive noise. In this paper, we propose a modification of the GP algorithm, which enhances the convergence by incorporating more *a priori* information about the object and improving the initial estimate of the iterations. We call the new algorithm a Local-Field GP (LFGP) algorithm because, in dynamic imaging, changes are assumed to be confined to a limited region of the object.

2. THEORY OF THE LFGP ALGORITHM

The original dynamic image, $f(x)$, is assumed to consist of a dynamic region and a stationary region, $h(x)$, which is known *a priorily* from the reference image and may be represented as

$$h(x) = f(x).(1-R(x/a)) \tag{1}$$

Where a defines a symmetric dynamic region extending around the center of the object and R is a rectangular window. Because only the central part of the spectrum is acquired to reduce the acquisition time, the measured data, $G(w)$, is given by

$$G(w)=F(w)\,.\,R(w/b) \tag{2}$$

Where b is the bandwidth of the rectangular window, and $F(w)$ is the Fourier transform of the dynamic image, $f(x)$. Taking the Fourier transform to both sides of equation (1), we get

$$H(w)=F(w) - sinc(aw)\otimes F(w) \tag{3}$$

Where $\otimes$ is the convolution operator. In order to converge to the true image, the algorithm progresses by transforming the image from the k-space and the image space and vice versa. In the iterations, the available *a priori* information is used to constrain the image. Therefore, at the i^{th} step in the algorithm, the measured part of the spectrum, $G(w)$, replaces the estimated part, hence:

$$F_i(w)=F^c_{i-1}(w)\,[1 - R(w/b)] + G(w) \tag{4}$$

Where, $F^c_{i-1}(w)$ is the spectrum of the constrained image in the last iteration. On the spatial-domain side of the transform, only the rFOV of the dynamic image is updated, meanwhile the outer region is replaced by that of the reference image, hence

$$f^c_{i-1}(x)=f_{i-1}(x)\,[R(x/a)] + h(x) \tag{5}$$

Where, $f^c_{i-1}(w)$ is the image after applying the spatial constraints.

Taking the Fourier transform to both sides of (5), and substituting in (4), we obtain

$$F_i(w)=\{ sinc(aw)\otimes F_{i-1}(w)+H(w)\}[1 - R(w/b)]+G(w) \tag{6}$$

In order to simplify the derivation, we need to define the following orthogonal operators

$$P_a = sinc(aw)\otimes,\; P_b = R(w/b),\text{ and } Q_b = (1 - P_b). \tag{7}$$

Where P_a projects any given signal onto the subspace of finite-extent signals, P_b projects onto the subspace of band-limited signals, and Q_b is a projector onto a subspace of signals whose frequency components vanishes for $|w| < b$.

Using the operators defined above, equation (6) becomes

$$F_i(w)= Q_b\,\{P_a\,F_{i-1}(w)+H(w)\} + G(w) \tag{8}$$

Substituting from equations (2) and (3) into (8), then

$$F_i(w)=Q_b\{P_a F_{i-1}(w)+ F(w) - P_a\,F(w)\}+ P_b\,F(w) \tag{9}$$

Or,

$$F_i(w)=Q_b\,P_a\,F_{i-1}(w)+ F(w) - Q_b\,P_a\,F(w) \tag{10}$$

For equation (10), i starts from 1, and $F_1(w)$ is taken as

$$F_1(w)= F(w) - Q_b\,P_a\,F(w) \tag{11}$$

Then, the recursion in equation (10) can be given by

$$F_i(w)=\sum_{r=0}^{i-1}(Q_bP_a)^r(I-Q_bP_a)F(w) \tag{12}$$

Or,

$$F_i(w)=\sum_{r=0}^{i-1}[(Q_bP_a)^r-(Q_bP_a)^{r+1}]F(w) \tag{13}$$

The above power series summation reduces to the following

$$F_i(w) = F(w) - (Q_bP_a)^i F(w) \tag{14}$$

From Von Neuman's alternating projection theory [6], given two orthogonal operators Q_b and P_a that project a signal $F(w)$ on two subspaces **Q** and **P,** then

$$\lim_{i\to\infty}(Q_bP_a)^i F(w) = L \tag{15}$$

Where L is the projection of $F(w)$ onto a closed subspace defined by the intersection of **Q** and **P**. However, according to the definition of Q_b and P_a in equation (7), the intersection of **Q** and P is the null space. Therefore, as i grows without bound in equation (14), the term $(Q_bP_a)^iF(w)$ vanishes. Hence, $F_i(w)$ tends to $F(w)$ and convergence to the true image is guaranteed for the LFGP algorithm.

3. IMPLEMENTATION OF THE LFGP ALGORITHM

A typical MRI acquisition scheme takes little time to acquire a complete row of data. However, it has to wait for a relatively long period, called Time-to-Repeat (TR), to acquire another row. This allows the atoms to return back to their relaxation status so that when re-excited a significant signal is acquired. Therefore, reducing the number of the acquired rows is useful to increase the scanning temporal resolution. However, it causes degradation in the spatial resolution in the columns direction. Therefore, by taking the inverse Fourier transform of the rows, individual columns can be subjected to the LFGP algorithm to extrapolate the missed samples. Alternatively, the LFGP can be directly applied to the whole image. This is achieved by using a 2-D rectangular window, as well as a 2-D Fourier transform.

Implementation of the LFGP algorithm starts by acquiring a reference image, in which we assume that only a region of interest undergoes dynamic changes. Next, the following algorithm is used to reconstruct the dynamic image:

1) Acquire the central part of the k-space of the dynamic image.

2) Apply the keyhole to reconstruct the k-space of the initial dynamic image $F_1(w)$ and let i=1.
3) Fourier transform the resultant k-space to obtain $f_i(x)$.
4) Replace the image regions outside the local field of the dynamic changes with their counterparts in the reference image to obtain $f_i^c(x)$.
5) Get the Fourier transform of $f_i^c(x)$ to obtain the spectrum $F_i^c(w)$
6) Replace the central part of F_i^c with that of the measured part of the k-space to obtain F_{i+1}.
7) Go to step 3.

The termination criterion, in the above algorithm, can be either a predefined number of iterations or when the error gets below certain threshold.

4. RESULTS

The ability of the LFGP algorithm to reconstruct simulated images is compared against that of the keyhole technique. A reference image of resolution (64x64) is first reconstructed as shown in figure 2. Dynamic changes are introduced into a central region that constitutes 30% of the reference image to simulate a dynamic study. This is illustrated in figure 3.

In figures 4 and 5, the dynamic image is reconstructed from the central 14 lines of the k-space using the keyhole and the LFGP techniques, respectively. The latter was used for only 10 iterations. It is apparent that LFGP, unlike the keyhole, is capable of tracking the change in the sharp edges, which means changes in the high frequency components of the image. This result may be further clarified if we examine a cross-section in the image. For this purpose, the central column of the actual dynamic image, and the reconstructed images in both figures 4 and 5, are plotted in figure 6. It is apparent that the image reconstructed by the keyhole, unlike the one reconstructed by

dynamic changes.

As expected, the error in the reconstructed image using the LFGP algorithm decreases with the number of iterations as shown in figure 7. The error curve indicates that acceptable results are obtained in less than 10 iterations, a process which takes less than 0.5 second on a P-II, 400 MHz, with 128 MB RAM.

For purposes of comparison, the region outside rFOV, which is assumed known *a priorily*, is not included in the error calculations either for the keyhole or the LFGP techniques. Figure 8 shows that the choice of the initial image in the LFGP affects its performance. Using a keyhole reconstructed image as the initial solution for the iterations results in a significant reduction in the number of iterations needed to reach the true image. On the other hand, starting from a zero-padded spectrum, which is the common start in the generic GP algorithm, requires much more iterations to reach the same results.

5. DISCUSSION

The assumptions associated with the keyhole technique lead to inaccurate results in situations when the dynamic changes include the high frequency components of the object. Another problem is the discontinuity occurring when concatenating regions from the different k-spaces of the reference and the dynamic images. Furthermore, the keyhole technique does not utilize the assumption that the dynamic changes are confined to a small FOV. On the other side, the rFOV technique acquires data that is needed to reconstruct a reduced FOV, rather than the entire FOV. Since imaging a smaller FOV corresponds to under-sampling in the k-space, the number of the acquired lines, and hence the acquisition time, is reduced. However, this causes the changes outside the rFOV to be wrapped inside it.

Unlike the keyhole technique, the proposed algorithm does not assume stationary high frequency components but tries to estimate them based on image information outside the local field of the dynamic changes. Moreover, in the LFGP algorithm, the assumption of local-field dynamic changes is applied, as a constraint, to the image space. Therefore, unlike the rFOV technique, there is no under-sampling of the k-space thus aliasing is not a problem.

Although less than 10 iterations were sufficient to reconstruct the image in figure 5, we believe that more iterations may be required for reconstructing real images.

When the entire object undergoes dynamic changes, LFGP reduces to the usual GP algorithm, which only assumes finite extent of the object. However, in this case, the initial solution is better and hence fewer iterations are needed.

6. CONCLUSION

In this work, a modified GP extrapolation algorithm is used for MRI dynamic imaging to improve the spatial resolution of reconstructed images. The proposed algorithm avoids the imperfections associated with the assumptions of both the keyhole and the rFOV techniques. Moreover, the extrapolation has the effect of smoothing out the k-space discontinuity resulting when the keyhole technique is used. The results show that the proposed technique enhances the image resolution even when the dynamic changes include the high frequency content of the k-space. The future work includes the verification of the LFGP algorithm by applying it on real MRI dynamic images.

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7. REFERENCES

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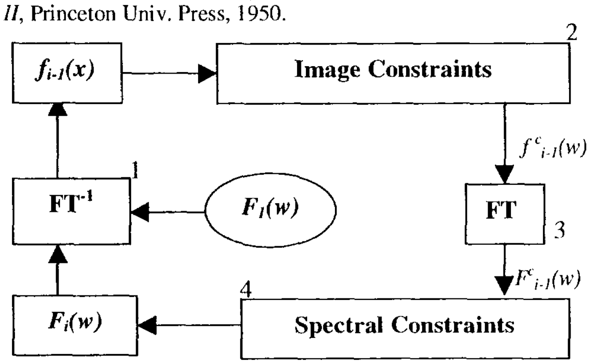

Figure 1. The generic GP algorithm, with the steps of the iterations indicated.

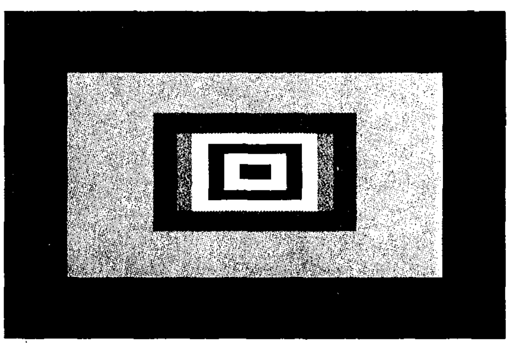

Figure 2. The reference image.

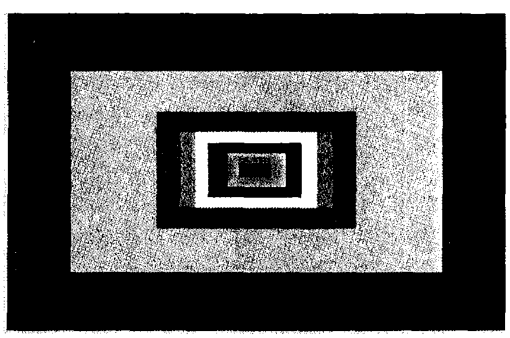

Figure 3. The Dynamic image.

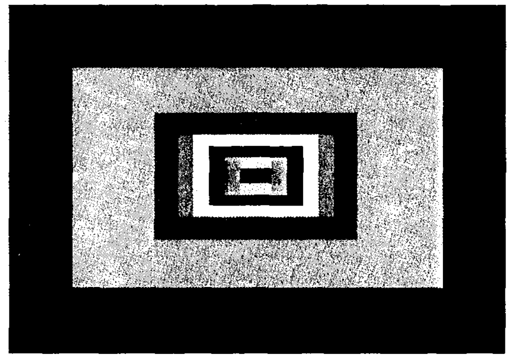

Figure 4. The dynamic image using keyhole reconstruction.

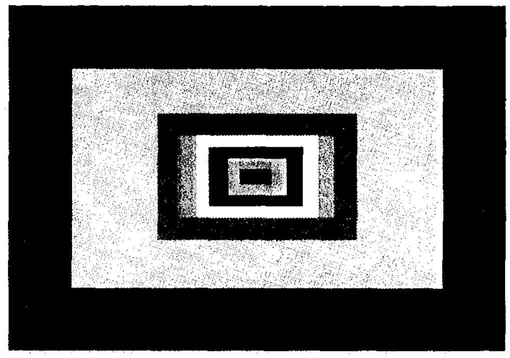

Figure 5. The dynamic image using LFGP reconstruction (10 iterations).

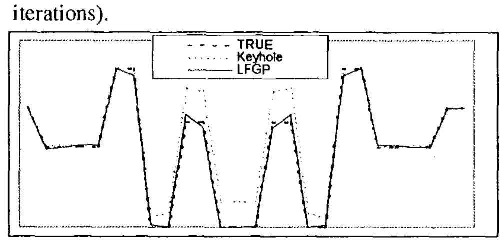

Figure 6. Central column of the images in figures (3,4, and 5)

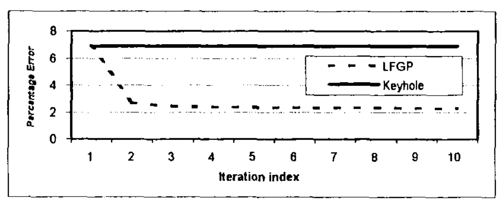

Figure 7. The error curve of the LFGP iterations. The fixed error of the keyhole is presented for comparison.

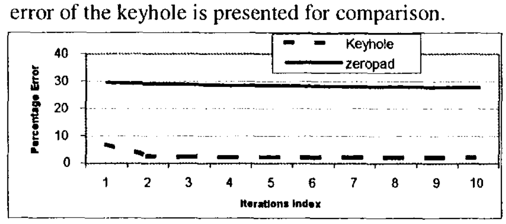

Figure 8. Iteration errors, using the Keyhole image, and a zero-padded image as the starting step of the algorithm.