THEORY OF TRUE-VELOCITY DUPLEX IMAGING USING A SINGLE TRANSDUCER

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ABSTRACT

In this paper we study the B-mode/Doppler duplex imaging problem. We begin by looking at the problem of determining the true velocity vector only. We develop a model for the power spectrum of a signal reflected by a line of point scatterers with a Poisson distribution. We show that with circularly symmetric apertures it is possible to use the expression of that power spectrum to determine the true velocity vector from a single excitation and two measurements. We also describe an illumination configuration that guarantees that the velocity estimation process is range-invariant. We conclude the paper by studying the problem of simultaneously estimating the range and true velocity of a flow. In particular, we show that this problem is completely characterized by a generalized range-2-D Doppler ambiguity function that depends on the excitation signal and the transducer geometry.

1. INTRODUCTION

Current Doppler ultrasound flowmeters are based on the classical Doppler frequency shift equation:

$$f_d = f_r - f_i = -\frac{v}{c}\left(\cos\phi_i + \cos\phi_r\right).f_i \qquad (1)$$

Here f_d is the frequency shift, f_r is the return frequency, f_i is the transmitted frequency, v is the target velocity, c is the speed of sound in tissues, and ϕ_i and ϕ_r are the angles between the direction of motion and the transmitter and the receiver axes respectively. According to this equation, the velocity of a moving target can be derived from the frequency shift of the return signal given the incident frequency, the wave phase velocity and the precise geometry of the problem. If we use the same transducer as both the transmitter and the receiver, we need only to define one angle since ϕ_i and ϕ_r in the Doppler equation will be the same in this case. As a result, if the path of the moving target is unknown, the above Doppler equation can only detect the projection of the velocity vector onto the direction of the transducer axis. Hence, if we want to obtain the magnitude of the complete velocity vector, we have to use at least three frequency shift measurements from three independent spatial locations to be able to estimate the three spatial velocity components. Although this has been the approach of many authors, the complexity of this technique have hampered its application in practical systems.

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One of the important constraints in using the Doppler equation is that the incident field must be a plane wave. This condition can be justified for many applications such as radar imaging where the target is far enough from the antenna. On the other hand, for practical medical ultrasound applications, this assumption no longer holds. This can be easily shown if we look at the angular spectrum decomposition of the ultrasound field at various ranges. A unidirectional plane wave requires an aperture that is infinite in extent and is therefore practically unrealizable. Hence, any finite aperture should be expected to return a spectrum of frequency shifts that corresponds to both the plane wave content of this aperture and the target velocity profile. This causes the well-known geometric broadening effect and is usually regarded as one of the major source of inaccuracy in the velocity estimation process. Many authors have investigated this effect along with the transit time effect on the Doppler spectrum [1]. According to their results, the received Doppler spectrum depends on both the axial and the lateral components of the flow. Hence, the estimation of two-dimensional flow maps is theoretically possible using a single transducer provided that the decomposition of the two components of the flow is feasible. This was the basis for the theory of estimating the transverse component of a Doppler flow developed for some special ultrasound fields [2]. This theory suggested that the geometric broadening effect can be used to calculate the lateral component of the flow by a formula that is similar to the classical Doppler equation for the axial flow. This formula can be applied well to flows in the far-field or in the focal plane of a focused transducer and provided that the flow crosses the axis of the transducer. The main difference between the new formula and the classical Doppler equation is that the formula uses absolute bandwidth measurements of the return spectrum instead of the Doppler shift.

In spite of the promising experimental results demonstrated with this theory, the strict conditions that must be met for the theory to apply have limited its practical application in practical medical imaging systems. Example of such conditions is that the flow line has to cross the transducer axis. Moreover, for the case of focused transducer arrangement, the flow has to cross that axis exactly at the focal plane. This is quite difficult to guarantee in practical situations where the flow location and direction are not available. Also, in case of circularly asymmetric apertures, e.g., a rectangular transducer, the flow was assumed to lie within a plane that is parallel to one of the rectangle sides while containing the transducer axis. Other problems as-

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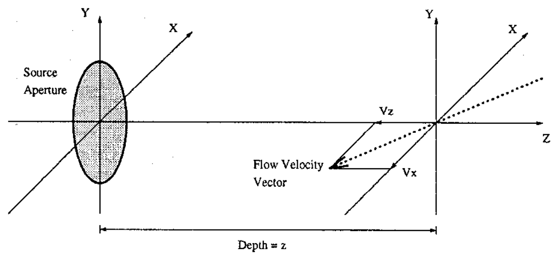

Fig.(1) Graph of the imaging problem.

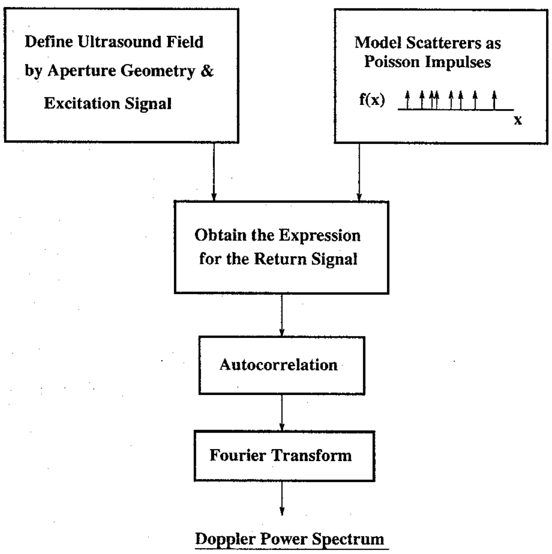

Fig.(2) Model derivation steps

sociated with the difficulty in measuring the narrow bandwidth variation specially from the far field configuration and its range dependence.

In this work, we present a general model to describe the combination of transit-time and geometrical broadening effects on the Doppler return power spectrum from an oblique flow. In the special case of narrowband signal and circularly symmetric aperture, we show that the magnitude of the complete velocity vector can be theoretically obtained by doing only one excitation and taking two measurements from the return power spectrum. Also, we establish the range and azimuth bandwidth shift-invariance theorems for this configuration. Then, we show that the Duplex image formation process can be described in terms of a generalized range-azimuth-velocity ambiguity function of excitation signals and transducer geometry.

2. VELOCITY ESTIMATION MODEL

In this part, we develop the velocity estimation technique based on a model for the power spectrum of the return signal from an oblique flow. Assume that we have a line of moving Rayleigh scatterers in front of an arbitrary aperture as shown in fig.(1). Under the conditions defined in [3] and given the aperture geometry and the temporal excitation, we can proceed to derive the formula for the Doppler power spectrum as summarized in the block diagram of Fig.(2). The resultant formula for the power spectrum for this model is given by:

$$\begin{aligned}\Phi_{RR}(\omega) &= \frac{2\pi\delta(\omega)\lambda'^2\sigma_s^2}{\left(v_x\cdot(1+2\frac{v_z}{c})\right)^2}\left|U\left(\frac{\omega}{v_x}\right)*S\left(\frac{\omega}{1+2\frac{v_z}{c}}\right)\right|^2_{\omega=0} \\ &+\frac{\lambda'\sigma_s^2}{\left(v_x\cdot(1+2\frac{v_z}{c})\right)^2}\cdot\left|U\left(\frac{\omega}{v_x}\right)*S\left(\frac{\omega}{1+2\frac{v_z}{c}}\right)\right|^2 \qquad (2)\end{aligned}$$

Here $\Phi_{RR}(\omega)$ is the Doppler power spectrum, λ' is the Poisson model parameter which can be related to the hematocrit value (ratio of the volume of the formed elements to the total volume of blood), σ_s is the scattering cross section of the individual scatterers, v_x and v_z are the transverse and axial components of the flow, $U(.)$ is the Fourier Transform of the effective transmit-receive aperture at the depth of interest, $S(.)$ is the Fourier transform of the excitation signal, c is the ultrasound velocity in the medium. In most practical applications, the first term in the power spectrum formula representing the DC power vanishes. When a narrowband excitation is used, it can be shown that the axial flow component can be determined from the second term by measuring the frequency shift of the returned signal. Furthermore, if we are given the envelope of the excitation signal and the beam pattern, we can estimate the lateral component of the flow by measuring the bandwidth of the return power spectrum. Now note that if the aperture is circularly symmetric, the component of the flow that is perpendicular to the x-z plane will always be zero. Therefore, we can effectively obtain the full length of the complete velocity vector by measuring v_x and v_z as explained above.

3. SPACE INVARIANCE OF MEASUREMENTS

To separate the temporal and spatial parts of the power spectrum in the above model, we need to determine the spatial beam pattern $u(.)$ from the projection of the flow line on the range plane of interest. The beam pattern will generally be a slice of the two-dimensional beam pattern at the depth of interest. Similarly, the spectral broadening formula that yields the transverse velocity in the theory of transverse flow estimation requires that the flow pass exactly through the center of the beam. In practice, this condition will not necessarily hold true. It also follows from the well-known theory of Fresnel propagation [4] that the field in the far field and in front of a focused transducer is depth-dependent. This makes the design of the receiver even more complex. Consequently, our aim here is to develop a technique that guarantees that the velocity estimation procedure is space-invariant.

Given that the wave propagation under Fresnel approximation can be represented by a linear system with a range-dependent space transfer function of infinite support, any beam will keep its absolute angular spectrum bandwidth over all ranges. For infinite bandwidth beams, it can be easily shown that any finite bandwidth measure, such as the commonly used $3dB$ bandwidth, will not be range-invariant. As a result, we have to look for a method to generate ultrasound beams with finite absolute bandwidth. Such beams can be generated in principle using an exact Fourier transform symmetric lens configuration instead of the commonly

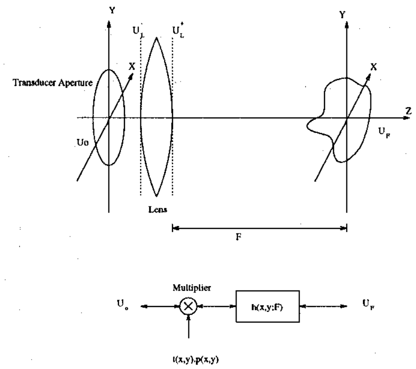

Fig.(3) Asymmetric lens configuration

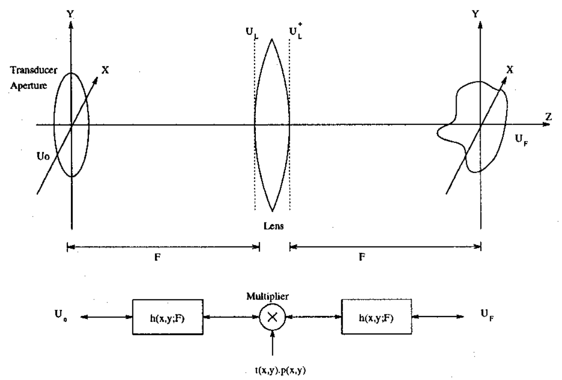

Fig.(4) Symmetric lens configuration

used asymmetric configuration [4]. These configurations are shown in Fig.(3)-(4). For the symmetric configuration, the beam in front of the lens will be guaranteed to have a finite bandwidth if the aperture in the front focal plane is finite. Hence, if this beam is used in our application, range and azimuth shifts of the flow will cause no change to the absolute bandwidth of the return Doppler power spectrum provided that Fresnel propagation holds.

A problem with the symmetric configuration that has to be addressed in practice is the vignetting effect associated with a finite extent lens. This effect has been investigated using computer simulations and the results indicate that this effect is quite minimal for practical depths/aperture sizes. The angular spectrum of an example of this configuration is shown in Fig.(5). As we can see, the angular spectrum is practically range-invariant.

4. GENERALIZED AMBIGUITY FUNCTION MODEL

Let us now consider the more general problem of duplex imaging or two-dimensional flow mapping. In this problem, it is required to estimate the two velocity components described above in addition to range and azimuth localization in the image plane. We show in this section that duplex imaging can be completely described by a generalized ambiguity function. We shall assume a uniformly homogeneous and attenuation-free medium between the transmitter and

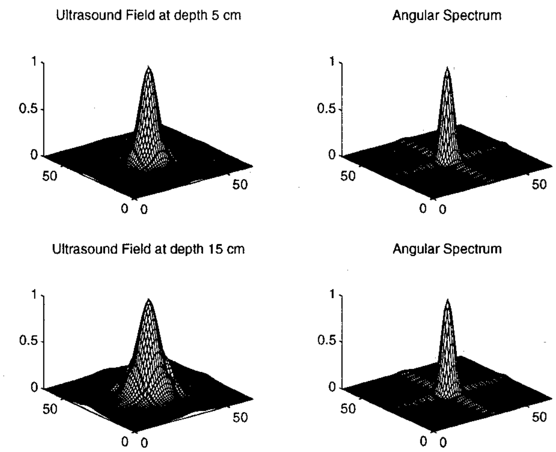

Fig.(5) Circular aperture with Gaussian apodization in the symmetric lens configuration.

the target. Furthermore, we assume that the transmitted signal is a complex exponential signal of the form,

$$s_t(t) = A \cdot exp[j\omega_c t] \quad (3)$$

By using a derivation similar to that found in Chapter 9 of [5] we find that the return signal $s_r(t)$ can effectively be modeled as

$$s_r(t) = A \cdot u(x) \cdot \tilde{b} \cdot exp[j\omega_c(t-\tau)] \quad (4)$$

where $\tilde{b}$ is a complex Gaussian random variable and x denotes the x-coordinate of the moving scattering cluster in the target flow. This model holds true provided that more than six scatterers are present in the cluster at location x. In the actual situation of RBC's in the blood stream, we expect to see a much larger number of scatterers, even under the most severe abnormally low hematocrit figures. The only minor exception to this is the very thin cell-free skimming layer in blood vessels.

If we further assume the reflection process to be frequency independent, then a transmitted signal $s_t(t) = A \cdot exp[j(\omega_c + \omega)t]$, will produce a received signal $s_r(t) = A \cdot u(x) \cdot \tilde{b} \cdot exp[j(\omega_c + \omega)(t-\tau)]$. Note that this assumption can still hold in our situation if we consider only small frequency variations around a certain large bias value such that the fourth power variation of the backscattering cross section with frequency in Rayleigh model can be neglected. We also assume that the reflection process is linear. Thus, if we transmit $s_t(t) = s(t)$, we receive, $s_r(t) = \tilde{b} \cdot u(x) \cdot s(t-\tau)$ [5].

Assuming a uniform velocity $\vec{v}$ that makes an arbitrary angle θ with the transducer axis, we can decompose the velocity vector into two components: axial and transverse as defined in the model. That is, $\vec{v} = |\vec{v}| \cdot (\sin\theta, \cos\theta) = (v_x, v_z)$. Hence, the received signal can be expressed as,

$$r(t) = \tilde{b} \cdot u(x + v_x t) \cdot s\left(t\left(1 + \frac{2v_z}{c}\right) - \frac{2z}{c}\right) + \tilde{w}(t), \quad (5)$$

where (x, z) are the coordinates of the moving scattering cluster and $\tilde{w}(t))$ is an additive noise assumed to be a zero-mean white bandpass Gaussian noise. The above equation can also be written as:

$$\tilde{r} = \tilde{b} \cdot \tilde{f}(t; x, z, v_x, v_z) + \tilde{w}(t). \quad (6)$$

Now, let the observation interval be infinite. We shall assume that the flow/location parameters are deterministic and of unknown values that we need to estimate. Then, up to a constant multiplier, the log likelihood ratio takes the form:

$$\ln \Lambda(x, z, v_x, v_z) = |\tilde{L}(x, z, v_x, v_z)|^2. \quad (7)$$

Here,

$$\tilde{L}(x, z, v_x, v_z) = \int_{-\infty}^{\infty} \tilde{r}(t; x^a, z^a, v_x^a, v_z^a) \cdot \tilde{f}^*(t; x, z, v_x, v_z) dt \quad (8)$$

where $\vec{\mathcal{M}}^a = (x^a, z^a, v_x^a, v_z^a)$ is the set of actual parameters that we need to estimate. The log likelihood ratio can be viewed as a continuous function of $\vec{\mathcal{M}} = (x, z, v_x, v_z)$. In particular, it may be written as:

$$\begin{aligned} \ln \Lambda(\vec{\mathcal{M}}) \quad &= |\tilde{b}|^2 \cdot \left| \int_{-\infty}^{\infty} \tilde{f}(t; \vec{\mathcal{M}}^a) \cdot \tilde{f}^*(t; \vec{\mathcal{M}}^a) dt \right|^2 \\ &+2 \cdot \mathcal{R} \left\{ \tilde{b} \cdot \tilde{n}(\vec{\mathcal{M}}) \int_{-\infty}^{\infty} \tilde{f}(t; \vec{\mathcal{M}}^a) \cdot \tilde{f}^*(t; \vec{\mathcal{M}}) dt \right\} \\ &+|\tilde{n}(\vec{\mathcal{M}})|^2. \end{aligned} \quad (9)$$

In the above expression, $\tilde{n}(\vec{\mathcal{M}})$ denotes an inner product between the noise $\tilde{w}(t)$ and $\tilde{f}(t; \vec{\mathcal{M}})$. The first term is due entirely to the signal and is the only term that would be present in the absence of noise. Hence, the resolution characteristics of the system are determined by the following integral

$$\phi(x, z, v_x, v_z) = \int_{-\infty}^{\infty} [u(x^a + v_x^a t) \cdot u^*(x + v_x t)] \cdot \left[s\left(t\left(1 + \frac{2v_z^a}{c}\right) - \frac{2z^a}{c} \right) \cdot s^*\left(t\left(1 + \frac{2v_z}{c}\right) - \frac{2z}{c} \right) \right] dt. \quad (10)$$

Note that the above function is very similar to the wideband ambiguity function with two independent parts for the range-axial velocity and the azimuth-transverse velocity ambiguities. The usual wideband ambiguity function is a special case of the above generalized function when u(.) is a constant or the velocity in the transverse direction is exactly zero. As a result, the usual techniques for improving the accuracy of the range-Doppler radar can be applied directly in this more general problem. For example, to obtain an improved axial velocity/range estimation, we have to send multiple independent excitation signals. On the other hand, to improve the transverse velocity/azimuth resolution, we have to use several independent apertures.

It should be noted that the above generalized ambiguity function model does not hold if the bandwidth of the excitation signal is too large. In that case, the fourth-power dependence of Rayleigh scattering from the RBC's in the diagnostic ultrasound frequency range plays an important role in the imaging process. It can be shown that the resulting generalized ambiguity has the same form as the one given above with the signals $\tilde{f}^*(t; \vec{\mathcal{M}})$ and $\tilde{f}^*(t; \vec{\mathcal{M}}^a)$ replaced by their fourth derivatives with respect to time. This minor change in the form of the ambiguity function can cause difficulties in the estimation process. When the transmitted signal is narrowband with a monophasic excitation envelope (e.g., Gaussian envelope), the ambiguity function has a single peak along the range variable z. On the other hand, when the excitation is wideband and the fourth-power dependence of Rayleigh scattering cannot be neglected, the resulting ambiguity function will have multiple subsidiary peaks along the range variable z. Moreover, axial velocity estimation may be affected in that case as well if it is based on frequency shift measurements of the spectral peak. On the other hand, the absolute bandwidth of the reflected signal is not affected by the fourth-power dependence of Rayleigh scattering. Therefore, the lateral velocity estimation procedure need not be modified when a wideband excitation is used.

The above discussion suggests the use of a preprocessing inverse filter for the returned RF signal when the excitation is wideband. The filter is similar to the one used in computed tomography but with a fourth order frequency dependence. The use of such a filter yields a more localized ambiguity function in the wideband case.

5. CONCLUSIONS

We have shown that it is possible to obtain the magnitude of the full-length velocity vector using one circularly-symmetric transducer. The technique is range-invariant if a finite angular spectrum beam is used, e.g., with the symmetric lens configuration. The estimation process resolution can be represented by a generalized ambiguity function that depends on the excitation signal and the beam characteristics at the depth of interest. This description suggests that although we can get an estimate of the velocity using one aperture, multiple apertures will be necessary if we need to reach a better accuracy in this estimate. Future extensions to this work include flow direction estimation, inverse problem in case of finite sample size, the effect of medium attenuation and inhomogeneity on the accuracy, and the experimental verification of the technique.

6. REFERENCES

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