Robust Multi-Component Diffusion Tensor Estimation Using Projection Pursuit Regression

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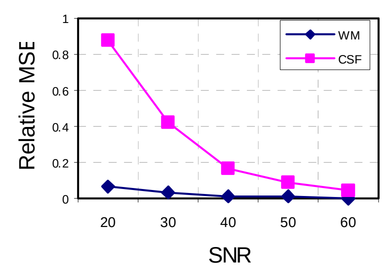
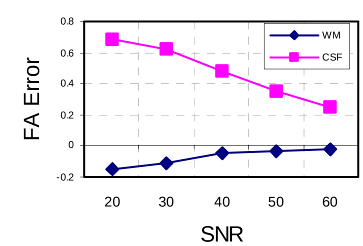
Introduction –The estimation of diffusion tensors in diffusion tensor imaging (DTI) is based on the assumption that each voxel is homogeneous and can be represented by a single tensor. As a result, estimation errors arise particularly in voxels with partial voluming of white matter or gray matter with cerebrospinal fluid (CSF) and voxels where fibers cross. Several authors have explored the possibility of solving for multiple tensors. Basser and Jones [1] analyzed the problem from the point of view of the number of unknowns and concluded that the solution is rather difficult due to the large number of unknowns and the nonlinearity of the equations. Tuch et al. [2], attempted to solve the special problem of fiber crossing by assuming the tensor eigenvalues in advance and using an iterative solution with multiple starting point to derive the angle between two fibers in their model. Several authors also attempted to use spherical harmonic representation of high angular resolution diffusion (HARD) to assist in the modeling process [3]. However, their approach was only helpful in eliminating some of the sources of artifacts in the DTI data but offered only a qualitative description of the model components. In this work, we develop a new solution strategy based on the well-known statistical method of projection pursuit regression (PPR). The new approach offers a fast and stable solution to the problem in its most general case by decomposing the problem into multiple 1D problems. This method was verified using computer simulations under various conditions of SNR and diffusion tensor composition.

Theory – Consider the problem of estimating the composition of a voxel with two components. In this case, the number of unknowns to fully describe the model is 14 (2 symmetric tensors and their magnitudes). Unlike the problem of estimating a single tensor, the equations here are nonlinear and therefore only iterative techniques can be utilized. We observe that the attenuation equation for each tensor resembles a sample of a 3D Gaussian function with a covariance matrix equal to the diffusion tensor evaluated at a point determined by the diffusion gradient direction at a radius equal to the square root of the b-value. Hence, the problem of estimating multiple tensors becomes one of estimating 3D Gaussian mixture model from samples determined by the diffusion gradient vector sampling. A robust statistical tool that allows the estimation of such models is the projection pursuit regression (PPR) or its equivalent learning using neural networks (PPL). Instead of attempting the solution in the 14D space of this problem, this method projects the problem into a number of 1D problems and then synthesizes the solution to the original problem from the 1D solutions. In our problem, the projection of the sum of two 3D Gaussian functions amounts to the sum of two 1D Gaussian functions with variance given by $\sigma_i^2 = [\cos\theta \quad -\sin\theta\cos\phi \quad \sin\theta\cos\phi] D_i^{-1} [\cos\theta \quad -\sin\theta\cos\phi \quad \sin\theta\cos\phi]^T$, for a projection direction given by angles θ and ϕ with the x and y axes respectively and where D_i is the diffusion tensor for one of the components. Moreover, the problem can be simplified even further by utilizing a sampling strategy in such a way to convert the problem into the sum of two exponentials. This problem is solved using a robust strategy in which the exponential decay constants are estimated using exhaustive search and the magnitude functions are estimated using linear system solution based on the choice of the decay constants. Given that the range of decay constants for human applications is rather limited, this strategy has superior speed to other strategies based on nonlinear least squares methods while offering the global solution to the problem. Once the 1D model is estimated, it can be used to provide an equation for each diffusion tensor separately as identified by its partial volume ratio. After sufficient equations are collected, the equations for each tensor are solved independently to obtain the tensor. In case of a 3-tensor model, the same procedure is followed at an additional computational cost that varies linearly with the number of tensors.





Methods – To verify the new method, Monte Carlo simulations were conducted to assess the accuracy of model estimation under different SNR and voxel tensor composition. The simulation parameters were as follows: the acquisition of a cubic volume of size 8×8×8 to fully cover one half of the 3D extent of the diffusion attenuation. The acquired data were projected onto a number of directions that uniformly sample the space. For convenience, these directions were taken to be similar to those used in HARD acquisition as 3- or 4-fold tessellated icosahedron vertices. Instead of selecting a few directions in the original PPR formulation, all directions are taken into consideration with a weighting corresponding to the model error. Also, within each 1D estimation procedure, a regularization step is implemented to verify that the partial volume ratio of all components is above a certain threshold value. This is necessary since it is likely that the component projections may have similar decay at some directions resulting in ill-conditioned solution.

Results and Discussion – The simulation results of a model composed of a white matter (WM) component in a random direction and CSF are shown. Notice that for SNR above 50 dB the estimation error of the tensors is below 10%. We notice also that the estimation accuracy of FA was also found to follow a similar trend. It is observed also that the WM component has less error than the CSF components at the same SNR. The model estimation procedure for each voxel was performed within an average of less than 1 sec, which is reasonable for practical purposes.

Conclusions – A new fast and robust method to estimate a diffusion mixture model is presented. The main advantage of this approach is the elimination of the dependence on a priori knowledge about the tensor composition, which allows more flexibility in practical applications. The new method was demonstrated using Monte Carlo simulations to be accurate and robust for SNR values above 50dB.

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References –

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