RF Pulse Design Using Combinatorial Optimization

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Abstract
The current RF pulse techniques do not take into account the practical implementation using finite resolution in both time sampling and amplitude discretization. Here, we formulate the problem of RF pulse design as a combinatorial optimization problem. The objective function is formulated to minimize the L1 difference between the desired profile and the computed profile using the exact solution to the Bloch equations. Two techniques were used to solve this problem, namely, genetic algorithms and simulated annealing. Both techniques are shown to provide better results than the classical Shinnar-Le Roux technique.

Introduction
The proper design of RF pulses in magnetic resonance imaging has a direct impact on the quality of acquired images. Several techniques have been proposed to obtain the RF pulse envelope given the desired slice profile (e.g., (1)). Unfortunately, these techniques do not take into account the limitations of practical implementation such as limited amplitude resolution. Moreover, implementing constraints for special RF pulses on most techniques is not possible. In this work, we propose an approach for designing optimal RF under theoretically any constraints. The new technique poses the RF pulse design problem as a combinatorial optimization problem and uses efficient techniques from this area to solve this problem. This ensures that the computed solution is still optimal under the practical implementation conditions.

Theory
Given the definition of the RF pulse, it is possible to compute the expected slice profile using the solution to the Bloch equations. This solution relies on using the analytical form for the slice profile from a single rectangular pulse of arbitrary magnitude given in (3). Keeping in mind the practical implementation of RF pulses in the form of piecewise constant envelope pulses (i.e., a sequence of rectangular pulses of arbitrary amplitudes), the output magnetization from one piece serves as the initial condition for the next. Hence, given any design for the RF pulse, the slice profile can be computed using this method. Given that the amplitudes of the RF pulses must be represented within a certain number of bits, the problem now becomes the one of finding the optimal combination of amplitudes that would give a slice profile closest to the desired. This problem description shows that this problem is indeed a combinatorial optimization problem. Using the rich literature of this area, the solution can be obtained efficiently and accurately. In this work, we explore two of the most prominent techniques in this area, namely, genetic algorithms and simulated annealing.

Methods
For genetic algorithms (GA), binary chromosome is used (3). The RF pulse is encoded into the chromosome as follows. The real RF pulse values are converted into discrete ones according to the resolution of the D/A of the MRI machine (12 or 16 bit for example). Every bit represents a gene. The most significant bits of all values are placed adjacent to each other, then the second most bits and so on until placing the least significant bits together at the end. Hence, the chromosome size equals the number of the RF pulse envelope values times the bit resolution (usually 12 or 16). The selection scheme was taken as biased roulette wheel. The probability of crossover was taken as 90% while that of mutation was 1% (3). On the other hand, the simulated annealing (SA) method used the Metropolis algorithm (4). We start with the RF pulse designed by Shinnar-Le-Roux (SLR) algorithm. A new solution is generated by adding a vector of random numbers to the vector containing the RF pulse values. The random vector should have small values to generate a new solution that is not far in cost function from the old solution. In both techniques, the solution obtained after each step is used to compute the slice profile by solving the Bloch equations (2). Then, an error measure is calculated for the difference between the response of this RF pulse and the desired response. This measure is used as the 1-norm (L1) of the difference vector since it gives equal weight to all error components.

Results
The proposed approaches were implemented to generate rectangular slice profiles at different flip angles. Figure 1 shows the comparison between the proposed methods and the SLR method. The new methods show a significant improvement over the SLR method and the SA is shown to provide a better solution.

Discussion
Imposing constraints on the solution using the proposed approaches is made such that the new solution is passed through a 'filter' that checks the validity of the new solution according to the required constraints. In very much the same way this was used for practical amplitude resolution, other conditions can also be included for specialized RF pulse such as adiabatic pulses.

References
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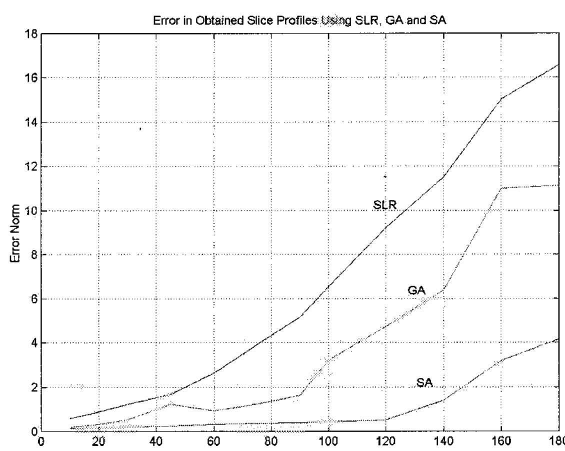

Figure 1. Comparison between error in slice profiles versus flip angle.