Gridding Using Spatially Variant Gridding Kernel

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Abstract
In this work, the mapping between the non-rectilinear samples and their rectilinear counterparts is given as the solution to a least squares problem. Given a particular k-space sampling, a training session is performed where by a matrix equation is formulated to map a local complete basis set of functions from the rectilinear format to the sampled points and the mapping matrix is obtained using SVD. This is achieved by sampling each of the training basis functions twice using both uniform and nonuniform sampling schemes. This mapping matrix is later used to reconstruct the data obtained using this trajectory.

Introduction
Non-rectilinear *k*-space trajectories are often used in MRI applications due to their inherent fast acquisition and immunity to motion and flow artifacts. However, fast image reconstruction techniques cannot be used unless the non-rectilinear data points are first resampled onto a rectilinear grid. Conventional gridding algorithms use a rigid Kaiser-Bessel window to approximate the process of Sinc function interpolation, which was shown to be optimal yet impractical (1). Even though these techniques have been used almost exclusively for image reconstruction when spiral sampling is performed, the variable sampling density within the k-space triggered the introduction of a number of new techniques during the past few years, e.g., the uniform resampling (URS) and the block uniform resampling (BURS) (2). The new techniques utilize reconstruction windows that are spatially-variant based on a few assumptions. The reconstruction results from such techniques were shown to be better than conventional techniques in some cases. On the other hand, their performance did not hold in some other cases such as with Lissajous trajectory sampling (3). In this work, we develop a more general formulation for the problem of resampling under the same assumptions as URS and BURS techniques. The new formulation allows the new technique to overcome the present problems with those techniques while maintaining a reasonable computational complexity.

Methods
The image space is decomposed into a complete set of orthogonal basis functions $f_i(k)$, i=1:N, with Fourier transformations denoted as $F_i(k)$. Each function is to be sampled twice, once with rectilinear trajectory and the other with nonrectilinear trajectory resulting in two vectors of samples F_i^r, F_i^{nr}. The required mapping matrix M_{NxN} is given by solving the set of the following linear equations:

$$F_i^r = M\, F_i^{nr} \quad , i=1:N$$

Each row in the matrix M is calculated at a time. This means that the above equations are solved for the j^{th} entry of the vectors F_i^r for every value of i to obtain the j^{th} row of M. The method used to do that is regularized SVD-based solver (4). In order to reduce the computational burden at the reconstruction time, a few non-rectilinear samples neighboring to the j^{th} point in F_i^r are used instead of the entire vector F_i^{nr}. This enables the computational complexity of the new technique to become comparable to that of conventional gridding techniques. The proposed technique is compared with URS algorithm for the 1-D case where undersampling was intentionally introduced to parts of the k-space of the image. The proposed shift-variant gridding kernel adapts this variation in the sampling density and proved to be better than the URS algorithm. Then, its variant called the block uniform resampling (BURS) technique was also compared to our technique for 1-D and 2-D cases using numerical phantoms.

Results
The new technique is applied to reconstruct images of 1-D and 2-D numerical phantoms as well as actual magnetic resonance imaging data acquired using a spiral trajectory.
The table below illustrates the reconstruction error as a function of both the sampling density factor (1-2), and the radius of the gridding kernel δk that defines the neighborhood used to compute the solution. The number of non-rectilinear points corresponding to this neighborhood is shown for clarity. In fact, the lower the number of points the faster the reconstruction process. The results show a superior performance of the proposed technique using a shift-variant gridding kernel. Noise effect on the performance is also tested and the proposed technique proved to be better than BURS algorithm.

Discussion
The results of this work demonstrated that the optimality of the Uniform Re-Sampling (URS) algorithm method is not guaranteed when the sampling density within certain *k*-space areas is reduced below the Nyquist threshold. We show that the results from the new technique are generally more robust against gross deviations from ideal sampling. Therefore, the new method has large potential in the new areas of imaging using variable density spirals and undersampled projection reconstruction acquisition strategies.

References
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δk	.625	.875	1.1	1.13	1.17	1.25	1.63	2.5
Density Factor	8	8	10	8	6	4	8	2
# Points*	5	7	11	9	7	5	13	5
New Method	3.5 %	3%	0.6 %	.56 %	.47 %	0.78 %	.23 %	.6%
BURS** (Δk=∞)	17.5 %	14.1 %	11.9 %	11.8 %	9.7 %	3.3 %	2%	.8%

*Number of points corresponding to δk at the given sampling density
** Δk=∞ corresponds to the best performance of the BURS technique.