Successive optimization for fast phase aberration correction

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ABSTRACT

We propose two new methods that allow the determination of the phase delays corresponding to phase aberration efficiently. We derive a new optimization methodology to compute the best compensation phase delays in successive steps. In particular, we start with an array consisting of one element with a specific excitation pattern. Then, another element is added and the dynamic receive delays are iteratively computed such that the obtained echoes are optimal in strength. A third element is added and the process is repeated. This process continues until all elements in the aperture are added. Hence, instead of solving the conventional N-dimensional problem of adjusting the delays of N elements together to achieve optimal characteristics, we transform the problem into the one of solving N-1 consecutive one-dimensional optimization problems. Given the fact that the set of available delay values is finite, the one-dimensional problem is shown to be a classical combinatorial optimization problem. The other technique based on Fourier transform tries to align signals based on information from a single frequency selected as the center frequency of the probe. This method is simple, computationally efficient and lends itself to real-time implementation. The proposed methods were implemented to correct real data from a resolution phantom and the results particularly indicate the potential of the second method.

Keywords: ultrasound imaging, phase aberration, simulated annealing, Fourier transform

1. INTRODUCTION

Ultrasound imaging is currently among the most commonly used diagnostic tools in medicine. Its versatile applications extend from basic anatomical imaging to flow mapping. If we consider the relatively low cost of ultrasound technology, we find that it is particularly interesting in today's world where the efficiency of health delivery is of prime importance. Therefore, a great deal of research activity in this area has been going on to extend its applications and lower its cost even further.

The basic idea behind pulse-echo ultrasound imaging is to send an ultrasound pulse through the body and wait to receive the echoes representing the reflection or scattering of the sent pulse from tissue interfaces within the imaged organ. Assuming the speed of ultrasound in the body to be equal to 1540 m/s, it is possible to reconstruct an image from the received echoes by mapping the echo delay into distance using this speed. Given the normal variations of this speed between individuals and even within the same organ of a certain individual, the measurements obtained by ultrasound imaging are prone to small errors as a result of these variations. Nevertheless, such deviations have not been considered a serious limitation to the technology and its use in different applications in general.

The resolution in ultrasound imaging depends on the different parameters used to acquire the image. The resolution is usually described in terms of two independent parameters representing the axial and lateral resolutions. The axial resolution depends mainly on the Q-factor of the probe, which determined the length of the radiofrequency (RF) excitation pulse. On the other hand, the lateral resolution is mainly a function of the characteristics of the excitation aperture such as aperture size and focusing accuracy. In order to improve the lateral resolution of a system, a large size aperture must be used with accurate dynamic focusing to adjust individually to every point in the ultrasound line. This

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can only be obtained using either array transducers with electronic focusing like for example annular array, phased array or linear array probes. Such systems consist of multiple receiving elements that receive the ultrasound echo independently from the field of view. Given their different spatial positions, echoes from a particular location within the field of view would arrive at different time for each element. By properly adding controlled delays to the elements of receive aperture, it is possible to match the arrival times of the echoes from a particular location in all elements such that the received echoes constructively interfere. This simple idea is the basis of dynamic focusing using array transducers, which is used on virtually all ultrasound systems having array probes. The difference between ultrasound systems relies on the way this is done. Observing that the accuracy of applying such delays is crucial to the quality of focusing, more expensive ultrasound systems tend to use smaller size elements with a more accurate delay circuit for each element. This is in contrast to low-end systems which conventionally use large-sized elements with fixed or limited range delays, which tend to cause poor focusing or blurring of the acquired image. Hence, the lateral resolution of an ultrasound system is determined by the accuracy of reconstructing the wave front by the receiver.

The way dynamic focusing technique is implemented on ultrasound systems is to again assume the speed of ultrasound imaging and use it along with the array transducer geometry to calculate the arrival times for each point in the ultrasound line. The relative delays between elements are adjusted in either a static or a dynamic manner to compensate for the arrival time differences between elements. This means that the variations of the speed of ultrasound from the assumed value will effectively cause errors in focusing, which manifest themselves as blurring and poor lateral resolution in ultrasound images. This problem is called phase aberration and it is encountered in many practical imaging situations that include breast imaging and abdominal imaging. This loss of image quality and degradation in resolution limits the ultrasound imaging technology to a significant extent. Therefore, a solution that enables the adaptive adjustment of focusing delays to maintain the resolution and image quality under practical conditions would be of great value to boost and extend the applications of this technology.

In this work, we propose two new methods based on global optimization and Fourier transform that allow the determination of the phase delays corresponding to phase aberration efficiently. We observe that the different elements in the imaging probe correspond to modulated versions of the same beam pattern in the focal area. Then, we derive a new optimization methodology to compute the best compensation phase delays in successive steps. In particular, we start with an array consisting of one element with a specific excitation pattern. Then, another element is added and the dynamic receive delays are iteratively computed such that the obtained echoes are optimal in strength. A third element is added and the process is repeated. This process continues until all elements in the aperture are added. Hence, instead of solving the conventional N-dimensional problem of adjusting the delays of N elements together to achieve optimal characteristics, we transform the problem into the one of solving $N-1$ consecutive 1-dimensional optimization problems. Given the fact that the set of available delay values is finite, the one-dimensional problem is shown to be a classical combinatorial optimization problem. Even though exhaustive search within a predetermined search range was found to be sufficiently fast, simulated annealing was used to perform the optimization to make the process even more efficient. The main advantage of the new methodology is its low computational complexity compared to conventional methods for the same final image quality. Hence, it has the potential for practical implementation for clinical ultrasound imaging.

2. PREVIOUS WORK

The area of phase aberration correction has received a wide range of research given its practical importance. Two main categories of solution approaches have been proposed in this area. The first is based on cross-correlation between signals from consecutive elements[1,2], while the other is based on using the speckle brightness as a quality factor[3]. Variants of these methods to improve the robustness of the correction as well as to speed up the calculations were proposed by several authors including parallel processing[4], dedicated hardware VLSI circuits[5,6], optimization of algorithms[7-12]. We note however that the best available computational complexity is still demanding and require fairly expensive hardware components to accomplish its minimum requirements for practicality. Hence, a technique that would improve on the accuracy and at the same time provides a simple computationally inexpensive solution would have a large potential in this important area of ultrasound technology.

3. METHODS

3.1. Time domain method based on simulated annealing

We observe that the objective function used for cross-correlation based techniques is not convex. An example for the sum-absolute-difference (SAD) function for real data is shown in Fig. 1, where this function was computed for a pair of consecutive RF lines as a function of their relative shift. As can be seen, a number of local minima exist within the search area. Given that the global minimum is the one sought to optimize the performance of the phase aberration correction, we realize that fast methods like gradient descent cannot be used in this application. Alternatively, two approaches can be used to find this point. The first relies on an exhaustive search within a range that is wide enough to contain the interesting local minima. This requires a considerable amount of computations, which mandate expensive implementation hardware and/or offline processing. This approach is the most commonly used approach for current implementations. It should be noted that the performance of the technique varies with initial search point as well as the range used. Moreover, the accuracy of time delay estimation is limited by the period in between RF sample points, which can be too coarse as compared to the range of time delay values reported in the literature. The other class of methods that can be used to find the global minimum is to use stochastic methods that use random search strategies that allow the global optimization to be performed in a more efficient manner than exhaustive search.

Another important consideration to take into account is the practical method used to generate the compensation delay in real ultrasound systems using digitally-controlled delays. In particular, for analog systems, the set of possible delay values is finite and limited by the number of bits used to control the delays as well as the delay circuit characteristics. On the other hand, digital systems are limited by the period of RF sampling (assuming simple delay implemented as a digital shift register in programmable logic device based systems or a shift operation in digital signal processing based systems). In both cases, we realize that our problem belongs to a subclass of optimization problems called combinatorial optimization. This means that our search space is limited and that our objective function is to find the global minimum that lies within the set of possible delay values that define the search space. A prominent tool to perform this task is the so called simulated annealing method.

The concepts of simulated annealing are based on a strong analogy between the physical annealing process of solids and the problem of solving large combinatorial optimization problems. Annealing is a thermal process for obtaining low energy states of a solid in a heat bath. It contains two steps. First, the temperature of the heat bath is increased to a maximum value at which the solid melts. Second, it is decrease carefully until the particles arrange themselves in the ground state of the solid. In the ground state the particles are arranged in a highly structured lattice and the energy of the system is minimal. The ground state of the solid is obtained only if the maximum temperature is sufficiently high and the cooling is done sufficiently slowly. Otherwise the solid will be frozen into a meta-stable state rather than ground state.

The Metropolis algorithm[13] is a numerical technique that simulates the annealing process. In this algorithm, given a current state i of the solid with energy E_i, then a subsequent state j is generated by applying a perturbation mechanism which transforms the current state into a next state by a small distortion (for instance by displacement of a particle). The energy of the next state is E_j. If the energy difference is less than or equal to 0, the state j is accepted as the current state. If the energy difference is greater than 0, the state j is accepted with a certain probability which is given by $\mathrm{Exp}((E_i - E_j)/(K_B.T))$, where T denotes the temperature of the heat bath and K_B is a physical constant known as the Boltzmann constant. The acceptance rule described above is known as the Metropolis criterion and the algorithm that goes with it is known as the Metropolis algorithm. We can apply the Metropolis algorithm to generate a sequence of solutions of a combinatorial optimization problem. For this purpose, we assume an analogy between a physical many-particle systems and a combinatorial optimization problem. This is based on the equivalencies that solutions in the combinatorial optimization problem are equivalent to states of a physical system, and that the cost of a solution is equivalent to the energy of a state.

The control parameter here is equivalent to the temperature. That is, if the energy difference is greater than 0, the state j is accepted with a certain probability which is given by the Metropolis criterion where $K_B.T$ is replaced by a single control parameter c. Initially the control parameter contains a large value. This means that most changes even those with large deterioration to the objective function will be accepted. The value of c is gradually decreased with each step until it approaches a very small value near 0. Then no deterioration to the objective function will be possible. This feature allows the simulated annealing algorithm to escape from local minima. Pseudo code for the simulated annealing implementation is given in Appendix A.

3.2. Fourier based phase aberration estimation

From the Fourier shift theorem, the time delay computed from the cross-correlation based methods corresponds to the group delay from the phase difference between the two signals of interest in the Fourier domain. This may imply that we can work within this parallel universe to compute the group delay and converge as the cross-correlation technique to the time delay corresponding to the present phase aberration. However, this is not practical for the following reasons:

a. It is not straightforward to compute the group delay from the phase difference between the two signals as a result of the presence of severe phase wrapping. The problem of phase unwrapping is a challenging one in practice and many techniques are not robust enough for our problem.
b. The computation of the Fourier transform for the all signals is computationally costly to start with given that we work with the significantly larger set of RF samples. We believe that this is the main reason why no research work was previously done where this technique is used.

Therefore, the basic application of Fourier transform to estimate the delay is considered not practical due to these reasons.

Here, we propose a simple, computationally efficient and non-iterative method to obtain a fairly accurate estimate for the time delay corresponding to phase aberration based on the Fourier transform method. The basic idea behind this technique is to align the signal components from all elements only at the center frequency of the probe. The proposed technique works as follows:

a. Compute the center frequency of the probe as the maximum of the Fourier transform of only one of the signals. We have experimentally observed that this center frequency is consistent between different elements and does not vary from one element to another. This amounts to an order of computations of $O(N \log_2(N))$ calculations, where N is the number of RF points. Note that we need to do that once per aperture unlike the conventional use of Fourier delay estimation discussed above, which requires one for each element in the aperture.
b. Compute only the frequency component corresponding to the center frequency for all elements using the basic discrete Fourier transform (DFT) formula. This amounts to a number of computations of complexity $O(N)$ per element.
c. Compute the phase differences between the computed center frequency components related to a chosen reference signal in the aperture. Here, we used a central element as our reference. We have observed minor changes in the performance when other reference elements, which lead us to believe that the selection of this reference is arbitrary. It should be noted however that we have generated all the results in this work based on our reference choice and that this effect of this choice should be addressed in a future study.
d. Estimate the relative delays as the phase differences divided by the center frequency. Note that the time delays here are computed in real format, unlike the discrete values provided by the cross-correlation based methods. We observe that such accuracy would require a much higher iterations in the first technique (or equivalently, a significant increase in the search space for exhaustive search based methods) while keeping a fixed cost of computations. We foresee that this property would simplify its implementation in practical digital beamformers as far as timing.

Given that the last two steps are of insignificant complexity, we observe that the needed total computational power is rather modest compared to existing techniques in the literature.

4. RESULTS AND DISCUSSION

The proposed methods were applied to correct real data obtained from the web site of the Biomedical Ultrasound Laboratory, University of Michigan. Although the techniques proposed were applied to several data sets, the data set that was used to generate the results in this paper is the one under “Acuson17”. The parameters for this data set are as follows: 128 channels, 13.8889 MSPS A/D sampling rate, 3.5 MHz transducer with 0.22mm element spacing, 2048 RF samples per line each represented in 2 bytes, and 8 averages. The data were acquired for a phantom with pins at different positions. We used the data to simulate a 48-channel beamformer on receive. The individual signals from the elements of an aperture location that coincides with one of the pins in the phantom was used for our experiments. Fig. 2 shows the computed SAD objective function for two lines within this aperture. Note the multiple minima that exist and observe that only one of them is actually the global minimum that is required to accurately compute the time delay corresponding to the existing phase aberration. The application of the simulated annealing method to obtain the solution leads to slightly improved computational time in our implementations. This is mainly the consequence of the fact that the computational performance of SAD-based techniques depends somewhat on the severity of the phase aberration problem in the data. Therefore, the only advantage of the simulated annealing based technique is its expected robustness under different conditions of the phase aberration problem.

Figs. 2 and 3 show an illustration of the magnitude and phase Fourier transform of the signals obtained from the 48 aperture elements respectively. The horizontal direction represents the frequency axis while the vertical axis corresponds to the element position in the aperture. Note that the magnitude information appears invariant with element position while the phase information shows quite a bit of variability. Fig. 4 shows a plot for the Fourier transform for the signal from one element while Fig. 5 is a zoomed version of the same figure to look at the range of significant power around the center frequency of the transducer. We have observed that the location of the peak of this curve is invariant for all elements within the aperture. Fig. 6 illustrates the calculated delays using the Fourier based method and Fig. 7 shows the performance of the new method compared to the uncorrected results under different selections of the center frequency around the one defined in the algorithm. We observe that the technique is generally robust against variations in this selection process within a wide range. This suggests the elimination of step a. of the method described above and use an approximate value from the known transducer characteristics for further reduction in computational complexity. It should be noted however that the optimal performance is achieved only with the true center frequency. Therefore, the compromise between these two properties can be flexibly addressed in this method. Note also that it is possible to measure the center frequency before the actual real-time scanning begins as a more accurate way of determining this important parameter.

5. CONCLUSIONS

We propose two new methods for phase aberration correction in ultrasound imaging. The first is based on a global search strategy to improve the robustness and efficiency of existing cross-correlation based methods. On the other hand, the second method is based on a new approach that works in the Fourier domain to align the signals at the center frequency of the probe. The second method is particularly interesting because of its new approach to the problem as well as its potential for real-time implementation. The proposed methods were applied on a real ultrasound data set and the results support the hypothesis of this work. Further work is needed to validate the proposed methods in clinical settings and to address the actual implementation on a scanner.

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APPENDIX A: PSEUDO CODE FOR SIMULATED ANNEALING

```
Procedure SIMULATED ANNEALING
BEGIN
      INITIALIZE (i start , C0, L0)
      K := 0;
      i := i start;

      REPEAT
       FOR L := 1 to Lk do
       BEGIN
          GENERATE ( j from Si );
          IF f(j) ≤ f(i) then i:=j
          ELSE
          IF  exp( (f(i) -f(j) ) / Ck ) > random [0,1) THEN  i:= j
      End
      k:= k+1
      CALCULATE LENGTH(LK)
      CALCULATE CONTROL(CK)
      CONTINUE UNTIL STOP CRITERION
END
```

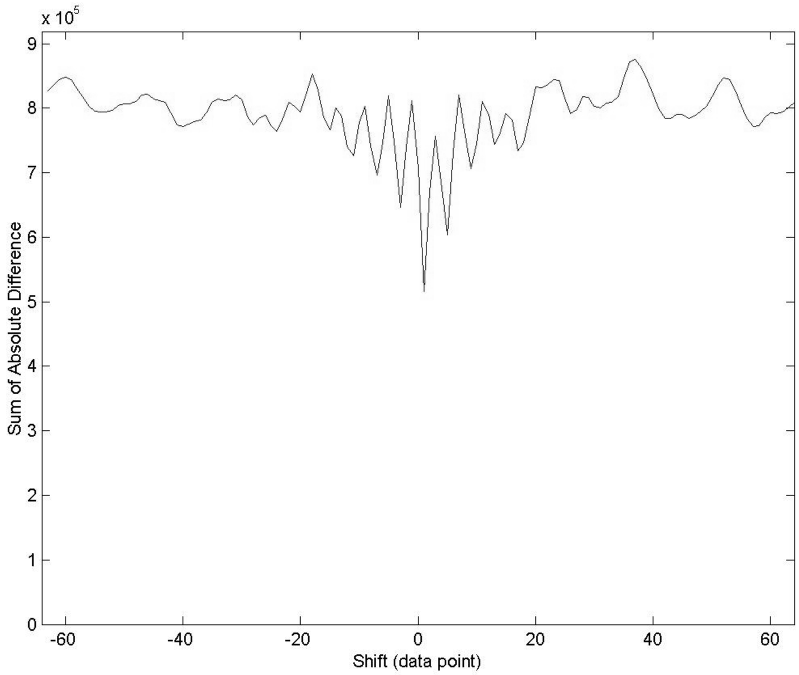

Figure 1. An illustration of an example of the sum-absolute-difference (SAD) objective function. Note that it is a non-convex function with many local minima.

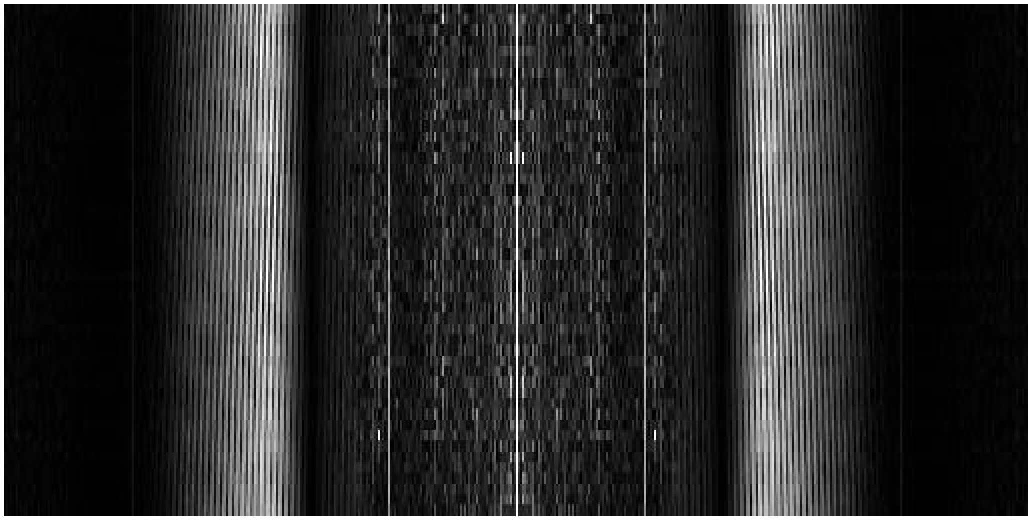

Figure 2. An illustration of the magnitude of the Fourier transform of a single element data. The horizontal direction represents frequency while the vertical is element number. Note the similarity between different elements.

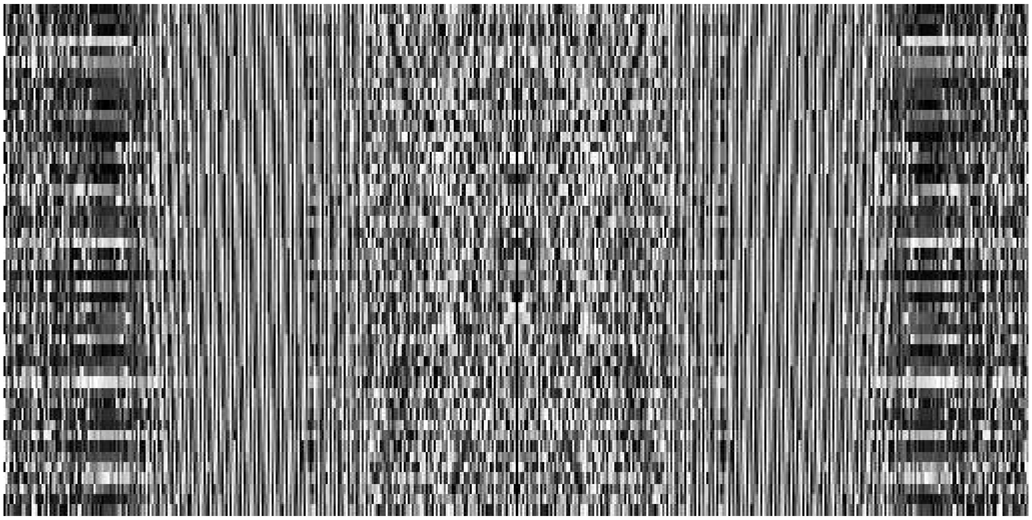

Figure 2. An illustration of the phase of the Fourier transform of a single element data. The horizontal direction represents frequency while the vertical is element number. Note the variations between different elements within the areas with significant magnitude.

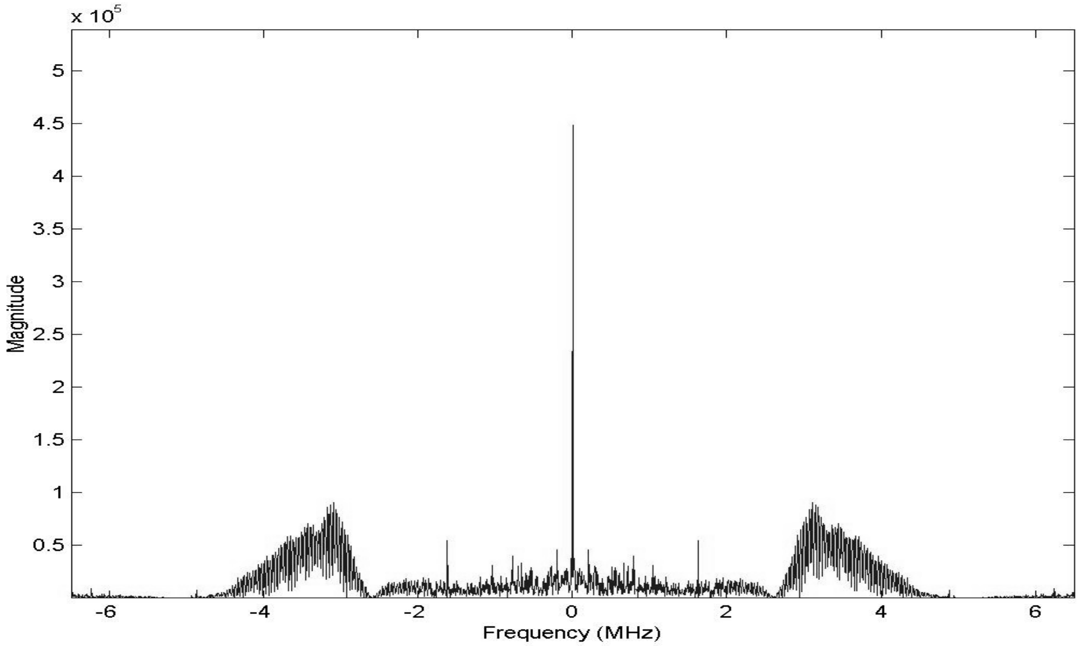

Figure 4. An illustration of the Fourier transform magnitude of the signal from a single element.

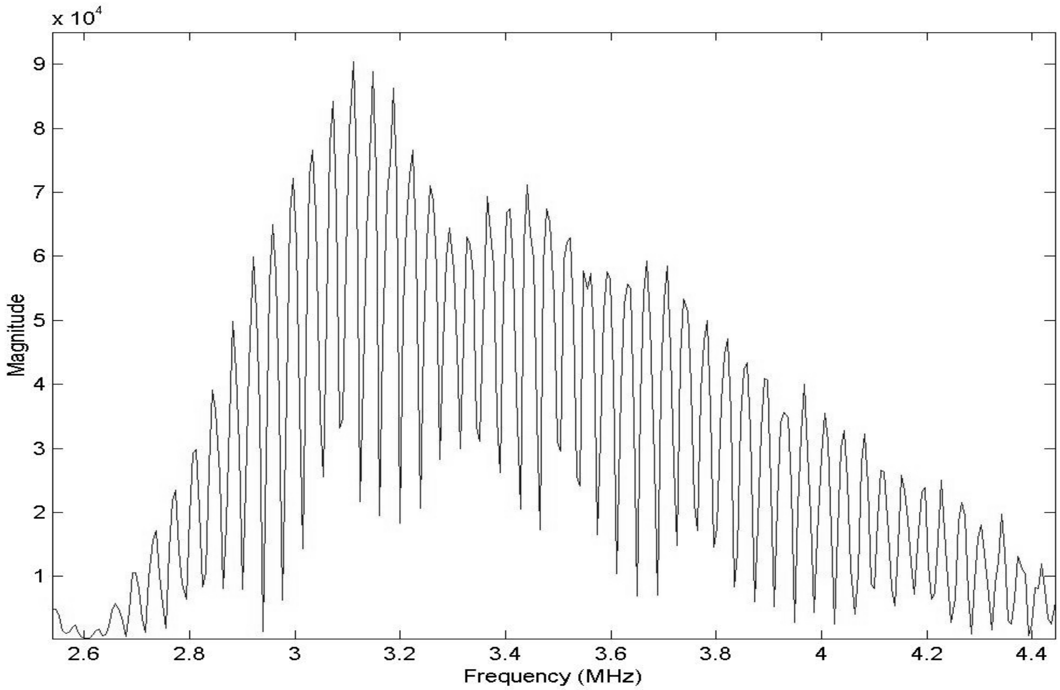

Figure 5. A zoomed version of Fig. 4 to show the range around the transducer center frequency.

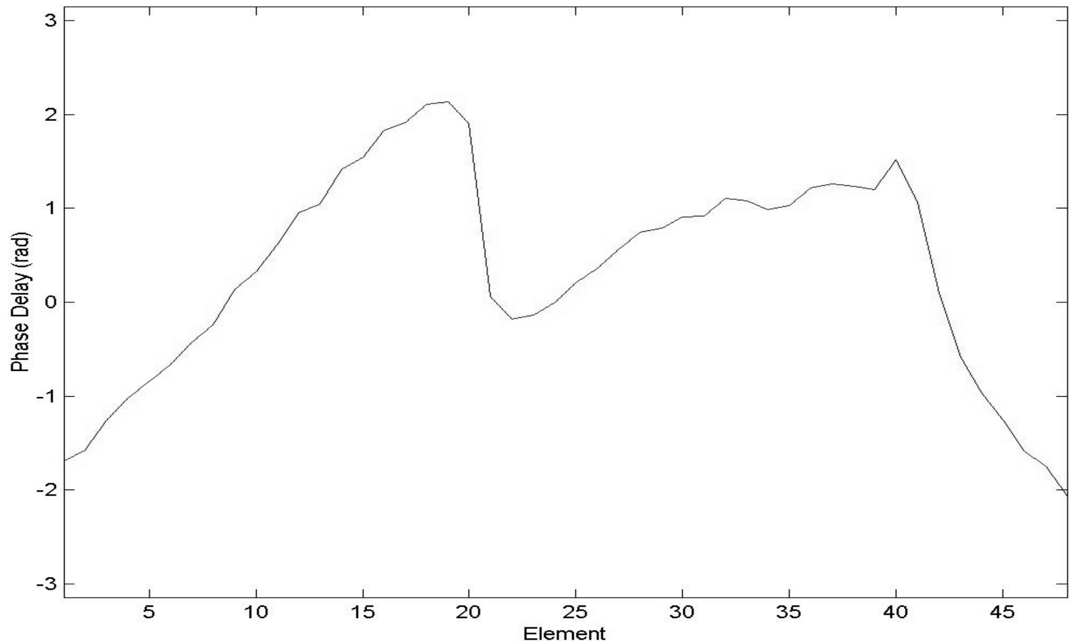

Figure 6. A plot of the estimated phase variations using the Fourier method.

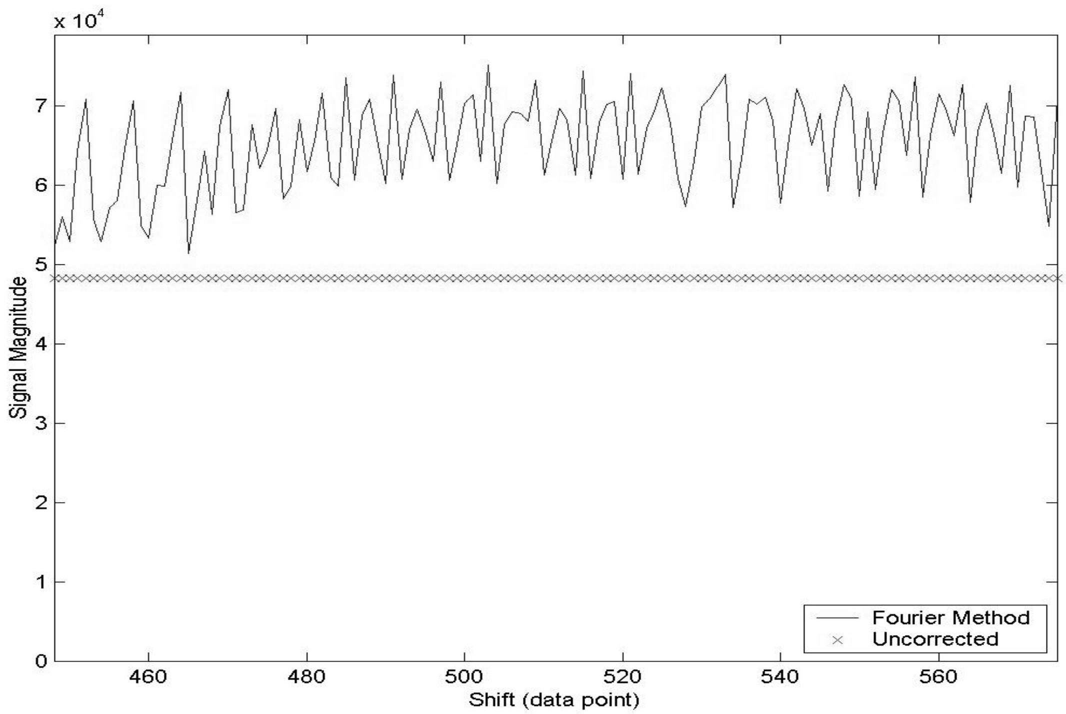

Figure 7. A plot of the maximum signal magnitude obtained using the Fourier based method under different selections of the center frequency. Notice that values within a wide range still provide better solution than the uncorrected.

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