COMPACT ANGULAR SUPPORT BEAMS FOR SPACE-INVARIANT VECTOR FLOW MAPPING

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ABSTRACT

In this work, we define a special class of compact angular spectrum beams (CASBs) that can be used to provide space-invariant estimation of the velocity vector from a single coplanar aperture. By space-invariance we mean both range and azimuth flow shift-invariance of velocity estimation. We analytically show that some of the non-diffractive beams, e.g., Bessel beams, are sub-classes of CASBs. We show that CASBs have infinite spatial support and thus, they cannot be generated in practice. We discuss finite support approximation of such beams. These approximations can be implemented using currently available annular and two-dimensional transducer arrays. We also describe the related class of spatially modulated CASBs. These beams provide range-invariant true-velocity estimation from frequency shift measurements.

1. INTRODUCTION

In conventional Doppler flow mapping techniques, the velocity values are assigned to different locations in the image based on frequency shift measurements according to the classical Doppler equation given by:

$$v \cos\phi = \frac{f_d c}{2 f_o}. \tag{1}$$

Here v is the true velocity magnitude, ϕ is the planar angle between the transducer axis and the direction of the velocity, f_o is the center frequency of the transducer, c is the phase velocity of ultrasound in tissues, and f_d is the measured frequency shift in the return signal. In this equation, f_o is known and c is usually assumed to be $1540 m/s$. On the other hand, the angle ϕ is not usually known and is difficult to estimate. As a result, the possible outcome from a single aperture velocity measurement configuration is only a projection of the complete 3-d velocity vector onto the transducer axis. Given the fact that different vessels within the same area have random spatial orientations, the different local velocity values within the obtained flow maps will have different references and hence, the diagnostic value of the technique is greatly compromised.

Some authors have observed that the assumptions under which the Doppler equation holds are not satisfied in conventional sonography. The Doppler equation is based on monochromatic plane wave excitations. The excitations used in radar imaging are monochromatic and correspond to plane waves. On the other hand, in the case of a finite focussed transducer used in pulsed mode Doppler for range gating, both assumptions are not satisfied. The lack of monochromaticity results in the so-called *transit-time broadening* effect, while the non-planar nature of the beam results in the so-called *geometrical broadening* effect. The broadening effect means that for a single velocity, the resulting Doppler spectrum will have a continuum of values around this value. Consequently, the above two broadening effects are considered as accuracy-limiting artifacts in the Doppler spectrum that should be eliminated in order to resolve flow gradients [1].

In spite of the difficulties introduced by geometrical broadening, some authors have suggested that this effect can indeed solve the problem of estimating the 3-d velocity vector. The idea is as follows: for a given finite transducer aperture, the resulting ultrasound beam is composed of a spectrum of plane waves of different orientations defined by the angular spectrum of the beam [2]. These different components interact independently with the moving object and thus undergo different amounts of frequency shift according to the classical Doppler equation. Since these plane waves have independent orientations, any 3-d velocity vector will be effectively observed from different directions and so, a solution is theoretically possible by solving the inverse problem given the angular spectrum beam. A problem arises with this approach if we consider the effect of spatially moving the trajectory of the moving object with respect to the transducer. Since the different plane wave components of the beam have different relative weights depending on the location in front of the transducer, the problem cannot be solved in general.

In this paper, we develop the necessary conditions on the imaging beam in order to obtain a space-invariant solution to the problem of true-velocity mapping.

2. DOPPLER POWER SPECTRUM MODEL

Consider the general case in which we observe the return Doppler spectrum from an oblique flow in front of an ultrasound transducer. Assume that the motion trajectories of RBCs inside the blood vessel in the case of laminar flow can be effectively represented as a collection of parallel straight lines in the direction of the vessel. The locations of RBCs

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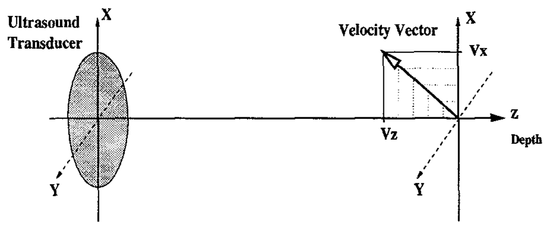

Fig.(1) Velocity estimation configuration

along any of these straight lines will be random and can be modeled as a Poisson impulses distribution. Then, up to a scale factor, the return power spectrum from a bandpass excitation to a single line of moving RBCs will take the following form [3]:

$$\Phi_{RR}(\omega) = \frac{\lambda' \sigma_s^2}{\left(v_x \cdot (1 + 2\frac{v_z}{c})\right)^2} \cdot \left| U\left(\frac{\omega}{v_x}\right) * S\left(\frac{\omega}{1 + 2\frac{v_z}{c}}\right)\right|^2 . \quad (2)$$

Here $\Phi_{RR}(\omega)$ is the Doppler power spectrum, λ' is the Poisson model parameter which can be related to the hematocrit value (ratio of the volume of the formed elements to the total volume of blood), σ_s is the scattering cross section of the individual scatterers, v_x and v_z are the transverse and axial components of the flow, $U(.)$ is the Fourier Transform of the effective transmit-receive aperture at the depth of interest, $S(.)$ is the Fourier transform of the excitation signal, and c is the ultrasound velocity in the medium. In words, the return power spectrum is essentially a function of the convolution of the frequency spectra of the excitation signal and the ultrasound field along the particular line of moving scatterers that produced the return signal.

If the excitation signal is chosen as a narrow-band signal, the axial flow component can in principle be obtained from measuring the frequency shift of the return power spectrum from the transmitted frequency. The transverse component can be measured from measuring the bandwidth.

3. ULTRASOUND PROPAGATION MODEL

According to the angular spectrum analysis of the scalar diffraction theory, it is possible to express any complex field distribution in terms of Fourier components. These components correspond to plane waves traveling in different directions. For example, for an aperture $a(x, y)$, the 2-d Fourier transform of the aperture $A(f_x, f_y)$ yields the angular spectrum after the substitution $f_x = \alpha/\lambda$ and $f_y = \beta/\lambda$ where α and β are directions cosines. It should be noticed that in order to have the magnitudes of α and β be less than or equal to 1, the magnitudes of f_x and f_y should both be less than or equal to $1/\lambda$. For finite apertures, this condition is not satisfied outside the circle of radius $1/\lambda$. The less interesting evanescent wave components appear from this part of the spectrum. As a result, the useful part of the angular spectrum can be defined to be within a finite circle of radius $1/\lambda$. Therefore, we will consider only this part of the spectrum in the following discussion.

If we assume that ultrasound propagation in tissues satisfies Fresnel propagation conditions, then the propagation of an ultrasound beam in space can be represented as a linear system with a space transfer function in the form [5]:

$$H(f_x, f_y; z) = e^{jkz} \cdot \exp\left\{-j\pi\lambda z\left(f_x^2 + f_y^2\right)\right\}. \quad (3)$$

Here, z is the propagation distance, k is the wave number, f_x and f_y are the spatial frequencies in the x and y directions, and λ is the wavelength. This analysis suggests that for a given incident wave packet, the transmitted wave packet at any point will necessarily have the same exact angular spectrum components phase-modulated as a function of the propagation distance (the range). Therefore, the theory suggests that the field from a finite aperture at any depth will have plane wave components moving in all possible directions with varying phases. This means that we can find two plane wave components in this expansion which travel in the same and the opposite directions to a transverse flow. As a result, the return power spectrum from a uniform velocity will theoretically have an absolute spectral bandwidth given by

$$BW_{abs} = \frac{2v_x f_o}{c}. \quad (4)$$

This bandwidth is not usually observed because the high spectral components usually have very small magnitudes that are masked noise. As a result, what we actually see is the effect of the strong part of the angular spectrum, which is usually the main lobe.

From the projection-slice theorem, the Fourier transform of the field along any radial transverse line is basically the projection of the angular spectrum at that plane along the direction of the line. Any lateral shifts of this transverse line will change the field along it by multiplying the angular spectrum by linear phase factors in f_x and f_y. Therefore, the field will be lateral shift-varying. This discussion suggests that it may not be practically possible to implement the velocity estimation method in the previous section. Any reasonable size flow is expected to contain a huge number of spatially-independent lines of moving scatterers. Given that the ultrasound field is space-dependent, it is not generally possible to measure the transverse velocity even from a uniform flow because differences in $U(.)$ result in an ambiguous effective bandwidth. In other words, the technique is impaired in practice by the fact that any shift of the moving line of scatterers will affect the outcome of the measurement.

4. CASB DEFINITION

In order to solve the space-variance problem, we suggest a class of beams which would in principle use the characteristics of the particular problem at hand to do space-invariant true-velocity mapping. This class is based on our power spectrum model and the properties of Fresnel propagation. We define compact angular support beams (CASBs) as those beams having the following properties:

1. their angular spectrum is real and vanishes outside a circle of radius $\mathcal{R}$ such that $0 < \mathcal{R} < 1/\sqrt{2\lambda z_{max}}$,
2. they maintain circular symmetry, and

3. they have an effective narrow-band time dependence.

The first condition ensures that $H(f_x, f_y; z)$ values will remain in the same quadrant such that the projection of the angular spectrum will have no zero crossings in the middle, which are likely to cause ambiguities in the estimation process. On the other hand, circular symmetry is important for space-invariance and narrow-band excitation is required for the Fresnel propagation model to hold. According to this definition, CASBs have several properties which we discuss as follows.

4.1. Space-Invariance of Angular Support

Any CASB with a minimum angular support defined by the radius $\mathcal{R}_o$ on the plane $\mathcal{Z}$ in space will propagate only to a CASB of the same support. This can be seen from the fact that the angular spectrum of the beam at any propagation distance $\mathcal{Z}$ in space is basically the multiplication of the beam at distance $\mathcal{Z}_o$ and the space transfer function $H(f_x, f_y, \mathcal{Z} - \mathcal{Z}_o)$.

4.2. Space-Invariance of Doppler Spectrum Absolute Bandwidth

Consider again the case of a single line of moving scatterers discussed above. Since all the values of the space transfer function of interest will be in the same quadrant, the projection of the angular spectrum will have no zero crossings. As a result, the computed power spectrum will have a range-invariant absolute bandwidth. There will be no ambiguity in determining this bandwidth in this case. Moreover, the spectrum maintains roughly the same shape in the ranges of interest. This is a desirable property since it allows us to design a suitable frequency shift measure based on detecting features in the shape of the spectrum.

For a laterally shifted line of flow, we can imagine that this line is a radial line of a shifted beam with a phase-shifted angular spectrum. Then, the support of the beam remains the same for a circularly symmetric aperture even though the effective beam itself changes. Hence, the absolute bandwidth of the beam is also lateral shift-invariant. To keep the spectrum shape, the shift should not exceed $4/\mathcal{R}$.

From these spatial shift-invariance properties, it can be seen that a transverse flow of uniform velocity that is composed of a large number of flow lines will yield a bandwidth that depends only on the lateral velocity component. Hence, transverse velocity measurement over the entire image plane is practically possible.

4.3. True-Velocity Estimation

It can be shown that the magnitude of the true-velocity can be obtained from a single aperture using CASB. The return power spectrum will contain two pieces of information which are sufficient to calculate the magnitude of the complete 3-d velocity vector; namely the absolute bandwidth and the frequency shift. The absolute bandwidth can be used directly to calculate v_x, while the frequency shift can be used to calculate v_z. Given the circular symmetry of the aperture, we can always choose our coordinate system such that any velocity vector will lie within the x-z plane. That is, the magnitude of the complete 3-d velocity vector can be obtained once v_x and v_z are known. Therefore, true-velocity magnitude maps can be generated in principle using CASBs from a single coplanar aperture.

4.4. Example: Bessel Beam as a CASB

The Bessel beam is one of the limited-diffraction beams. It is defined as [6]:

$$u_o(\vec{r}, t) = A \cdot J_o(\alpha r) \cdot e^{j(\beta z - \omega t)}. \qquad (5)$$

Here A is in general a complex constant, $u_o(\cdot, \cdot)$ represents the pressure at a given location and at a given time, $J_o(\cdot)$ is the zeroth-order Bessel function of the first kind, $\vec{r} = (r, \phi, z)$ represents a point in space, $r = \sqrt{x^2 + y^2}$, z is the axial distance, α and is a scaling factor that determines the beam width, ω is the angular frequency.

As one expects from a non-diffractive beam, the characteristics of the beam are invariant with the propagation distance. For example, the angular spectrum of the Bessel beam is range-invariant and is given by:

$$U_o(f_x, f_y) = \frac{A}{\alpha} \cdot \delta(\rho - \rho_o) \qquad (6)$$

where $\rho = \sqrt{f_x^2 + f_y^2}$ and $\rho_o = \frac{\alpha}{2\pi}$. Since the Fresnel free-space transfer function is circularly symmetric, it can be shown that the angular spectrum absolute bandwidth will not change with depth.

Consider now the case of transmitting a Bessel beam and receiving the return with a very small transducer. It can be shown that the returned spectrum from a general transverse flow passing through the origin of any range plane in a Bessel beam is given by:

$$\Phi(\omega_x) = \begin{cases} \frac{2A}{\alpha} \cdot \frac{\rho_o}{\sqrt{\rho_o^2 - \omega_x^2}} & : \ \omega_x < \rho_o \\ 0 & : \ \omega_x > \rho_o \end{cases} \qquad (7)$$

where ω_x is directly proportional to the lateral flow velocity. This can be proven from the projection-slice theorem by taking the projection of the above angular spectrum onto f_y and observing that f_x corresponds directly to ω_x. It can also be shown that the bandwidth of the returned spectrum of a general transverse flow passing through any range plane in a Bessel beam is lateral shift-invariant for a given lateral velocity. The spectrum shape itself can be shown to be the projection of the multiplication of the angular spectrum given above with linear phase terms in f_x and f_y. Since the angular spectrum has circular symmetry, this establishes that the Bessel beam is a CASB.

5. SPATIALLY MODULATED CASBS

Suppose now that we modulate a CASB by a given spatial frequency ω_m^x in the x direction. Then, the resulting beam will consist of two circles of radii $\mathcal{R}$. The separation distance between the centers of these circles is $2\omega_m^x$. Obviously, this beam will not be a CASB. Nevertheless, it will have some desirable properties that are worth discussing. The effect of this spatial modulation in the x-direction is to induce a frequency shift in the return signal that is dependent on the

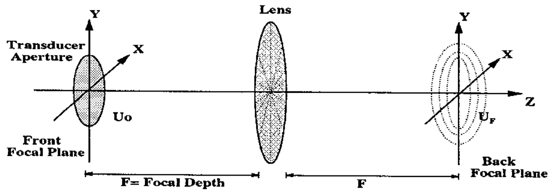

Fig.(2) Exact Fourier lens configuration

lateral velocity component in the same direction. The spatial modulation of this beam will be range invariant because of the range-invariance of the angular spectrum of CASBs. Hence, the resulting frequency shift is also range-invariant. Inducing another modulation in the y-direction enables us to measure the other lateral component via frequency shift measurements. Hence, range-invariant true-velocity measurement is possible in this case from frequency shift measurements.

6. IMPLEMENTATION OF CASBS

From their definition, CASBs have infinite spatial support on any plane in space. This is a simple consequence of the fact that their angular spectrum support can be visualized as an infinite support spectrum windowed by a finite circular window. Since the bounding circle of the CASB is less than $1/\lambda$, it can be shown that the spatial representation will have to be of infinite extension. This means that exact implementation of a CASB is not physically possible since all practical apertures have finite support. However, in the theoretical sense, these beams can be generated using an exact Fourier transform lens configuration as the one shown in Fig.(2). The usual transducer-against-lens configurations cannot produce CASBs because of the quadratic phase present in its spatial form.

Even though exact realizations of CASBs are not practical, it can be shown that effective approximations which possess the desirable properties of CASBs are realizable for ultrasound imaging ranges. These realizations are based on a finite-width approximations for the infinite field behind the lens in the exact Fourier configuration. In Fig.(3), we show a simulated example of a finite-Gaussian angular spectrum produced by a 2-d array of 16×16 square of transducers. It is clearly possible to generate CASBs in much the same way as non-diffractive beams which have a similar realization problem.

7. CONCLUSION

We have defined a special class of ultrasound beams that can be used to obtain space-independent true-velocity flow mapping. The advantages of this class over others include space-invariance velocity estimation accuracy and the ability to perform single-shot true-velocity estimation effectively. The well-known Bessel beam has been shown to be a member of this class. We have demonstrated by simulations the possibility of generating good approximations to those beams using 2-d transducer arrays or annular arrays. Finally, we discussed the related class of spatially modulated CASBs and their space-invariant true-velocity measurement using frequency shift measurements only.

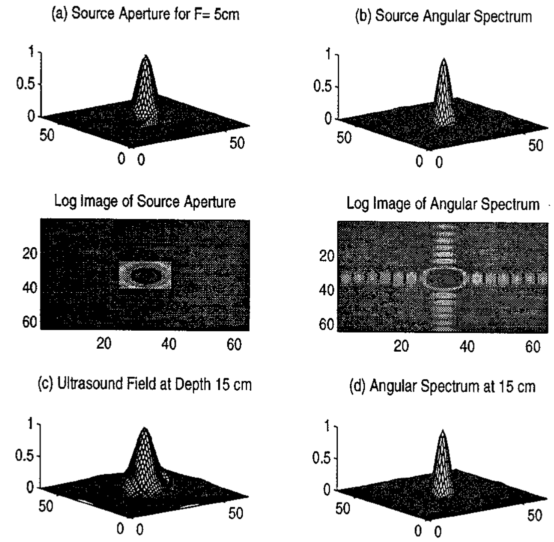

Fig.(3) Implementation of circular angular spectrum with Gaussian apodization

8. REFERENCES

[1] S.A.Jones, "Fundamental Sources of Error and Spectral Broadening in Doppler Ultrasound Signals", *Critical Reviews in Biomed. Eng.* , v.21, n.5, pp 399-483, 1993.

[2] D. Censor, *et al* , "Theory of Ultrasound Doppler-Spectra Velocimetry for Arbitrary Beam and Flow Configurations", *IEEE Trans. on Biomed. Eng.* , v.35, n.9, pp 740-751, 1988.

[3] Y. Kadah and A. Tewfik, "Space-Invariant True-Velocity Flow Mapping Using Coplanar Observations," *Proc. IEEE EMBC'95*, Montreal, Canada, Sept. 1995.

[4] Y. Kadah and A. Tewfik, "Theory of True-velocity Duplex Imaging Using a Single Transducer," *Proc. IEEE ICIP-95*, Washington D.C., Oct. 1995.

[5] J.W. Goodman, *Introduction to Fourier Optics* , McGraw-Hill, 1968.

[6] J. Durnin, "Exact Solutions for Non-Diffractive Beams, I. The scalar theory," *J. Opt. Soc. Am.*, v.4, n.4,pp. 651-654, 1987.