TRUE VELOCITY ESTIMATION USING THE CORRELATION TECHNIQUE

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ABSTRACT

The correlation technique has been shown to give a better accuracy velocity estimation than the conventional frequency domain methods. This technique requires two consecutive echo signals in addition to the flow angle to produce the magnitude of the velocity vector based on transit-time calculations. The effect of the lateral beam pattern on the results has not been sufficiently treated in the literature. In this work, we present a new generalized formulation of the correlation technique that incorporates both axial and lateral beam characteristics. We show that the location of the correlation function peak given in the literature is a special case of this model when the field is a plane wave. Also, we show that for other practical beam forms, the lateral beam form introduces an angle-dependent bias to the axial velocity measurements obtained with the classical formula. By properly choosing the source aperture and excitation signal, we derive a formula for lateral displacement estimation from correlation peak locations and magnitudes. This displacement can be used directly to estimate the lateral velocity component and allow the calculation of the complete velocity vector.

1. INTRODUCTION

One of the most important biomedical applications of ultrasound is its use in the detection, measurement and mapping of blood flow. In flow mapping, it is required to estimate the velocity value within each resolution pixel in the 2-d image plane. This can be done in general using one of two techniques: Doppler shift measurement and time domain correlation. In the Doppler shift based techniques, the movement of RBCs with blood flow in vessels causes a frequency shift in the return signal from a narrow-band excitation that is proportional to their velocity projection onto the transducer axis. Hence, in principle, measuring this shift is enough to calculate the velocity [1].

Nevertheless, due to the trade-off between the spatial resolution and the accuracy of velocity estimation, several excitations must be performed to obtain reasonable accuracy in both dimensions. This might not be feasible in many cases where flow pulsatility cannot be neglected and the temporal resolution of flow mapping is required to be high. For the time domain correlation techniques, the intuitive idea is to imagine RBCs in blood flow to be arranged into a random but fixed pattern that moves along the vessel with blood flow. Hence, the return signal at any time can be thought of as a snapshot of this pattern at that particular time. Consequently if we use two consecutive excitations such that edge effects can be neglected, the two snapshots obtained will represent exactly the same pattern with a slight shift corresponding to the distance traveled through the elapsed time period between the two excitations. Therefore, a correlation measure between the two return signals can indeed indicate this distance and hence, the velocity of the moving pattern. Given that the excitation signal does not have to be long as with the Doppler technique, it is clear that the trade-off between mapping resolution and accuracy in velocity estimation does not exist directly in this case. So, the time domain correlation technique is best suited for flow mapping applications [3].

Unfortunately, the present formulation of the correlation techniques take into account only one component of the velocity along the transducer axis. This means that the velocity estimates at different locations are in fact projections of the true velocities along different directions. Therefore, the resulting estimate cannot be used for accurate clinical interpretation. Also, the moving random patterns of scatterers encounter changing ultrasound field characteristics. These are likely to affect the outcome of the technique in much the same way as geometric broadening effects in the Doppler techniques. Therefore, a generalized model for the correlation technique that takes into account the ultrasound beam effects as well as the excitation effects is required to better understand this problem.

In this work, we present a generalized model for the output of the correlation technique. We consider a straight line of scatterers moving in a general direction in front of an ultrasound transducer. We demonstrate several special cases of this model to describe the current 1-d formulation and geometric shifts in the estimated velocity. Also, we observe that the lateral velocity component can be estimated under special conditions which we state. We show that these conditions are realizable using the current ultrasound technology. Finally we discuss the limitations of the technique in practical applications.

2. THEORY

Assume that a line of random scatterers with a Poisson impulses distribution is moving in a general direction in the imaging plane making an angle θ with the axial (z) direction. Choose the lateral direction x as the direction of the

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projection of the flow line on the transverse plane perpendicular to the z-axis at the depth of interest. Assume that the scatterers are identical perfect Rayleigh scatterers with scattering cross section σ_s and that they maintain a uniform velocity v throughout the effective period of insonification. Let the effective transmit-receive ultrasound field along the path of scatterers be denoted as $u(x, z)$, the temporal insonification window be $s(t)$, and the random scattering line process be denoted as $f(r)$. Then, the distribution of the scatterers across the flow line as a function of the distance to the observation point $r = \sqrt{x^2 + z^2}$ can be given by [2]:

$$f(r; x, z) = \sigma_s \cdot \sum_{n=-\infty}^{\infty} \delta(x - r_n \sin\theta, z - r_n cos\theta). \quad (1)$$

If we consider only small variations of the depth z around a given bias value z_o, the dependence of the lateral beam profile can be considered as an exclusive function of x in this domain. That is, $u(x, z) \approx u(x, z_o)$ in the region of interest. We shall call $u(x, z_o)$ as $u(x)$ for short. Then, the reflected signal from the process of insonifying the above process while moving at a uniform velocity $\vec{v} = (v_x, v_z)$ can be expressed as:

$$s_r(t) = \iint_{-\infty}^{\infty} u(x) \cdot f(x - v_x t, z - z_o + v_z t) \cdot s\left(t - \frac{z - z_o}{c}\right) dz\, dx \quad (2)$$

Hence, if the magnitude of the velocity vector is much smaller than the phase velocity of ultrasound in the medium, the received signal can be given in the form:

$$r(t) \approx \int_{-\infty}^{\infty} f(r) \cdot u(r \sin + v_x t) \cdot s\left(t + 2\frac{t v_z}{c} - 2\frac{r cos\theta}{c}\right) dr. \quad (3)$$

Now, let us look at the return from two consecutive excitations. Define the time variables t_1 and t_2 as the time axes in these two excitations and let T be the pulse repetition period. Hence, the return signals can be expressed as:

$$r_1(t_1) \approx \int_{-\infty}^{\infty} f(r) \cdot u(r \sin\theta + v_x t_1) \cdot s\left(t_1(1 + 2\frac{v_z}{c}) - 2\frac{r\cos\theta}{c}\right) dr \quad (4)$$

and

$$r_2(t_2) \approx \int_{-\infty}^{\infty} f(r - \Delta r) u(r \sin\theta + v_x t_2) \cdot s\left(t_2(1 + 2\frac{t v_z}{c}) - 2\frac{r\cos\theta}{c}\right) dr. \quad (5)$$

Then, the cross correlation can be obtained as:

$$R_{rr}(t, t+\tau) = \mathbf{E}\left\{r_1(t) \cdot r_2^*(t+\tau)\right\}. \quad (6)$$

Substituting from (4) (5) into (6), we obtain:

$$R_{rr}(t, t+\tau) = \iint_{-\infty}^{\infty} \mathbf{E}\left\{f(r_1) \cdot f^*(r_2 - \Delta r)\right\} \cdot u(r_1 \sin\theta + v_x t) u^*(r_2 \sin\theta + v_x(t+\tau)) \cdot s\left(t(1 + 2\tfrac{v_z}{c}) - 2\tfrac{r_1\cos\theta}{c}\right) \cdot s^*\left((t+\tau)(1 + 2\tfrac{v_z}{c}) - 2\tfrac{r_2\cos\theta}{c}\right) dr_1 dr_2 \quad (7)$$

From the properties of Poisson impulses distribution,

$$\mathbf{E}\left\{f(r_1) \cdot f^*(r_2 - \Delta r)\right\} = \lambda^2 + \lambda \cdot \delta(r_1 - r_2 + \Delta r) \quad (8)$$

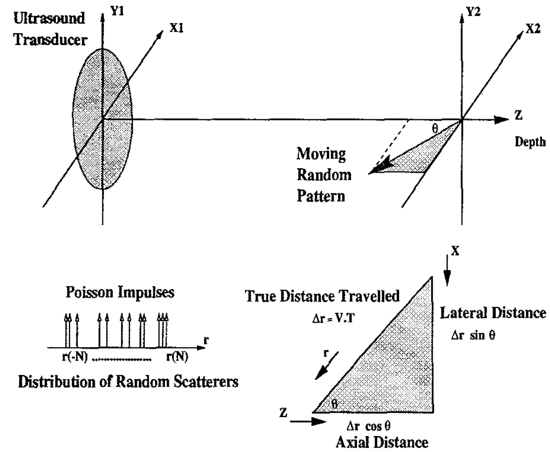

Fig.(1) Model for velocity estimation

where λ is the Poisson parameter which can intuitively be related to the hematocrit value from its definition. Substituting from (8) into (7), we get:

$$R_{rr}(t, t+\tau) = \lambda^2 \cdot G_1(t) \cdot G_1^*(t+\tau) + \lambda \cdot G_2(t, \tau) \quad (9)$$

where:

$$G_1(t) = \int_{-\infty}^{\infty} u(r \sin\theta + v_x t) \cdot s\left(t + 2\frac{t v_z}{c} - 2\frac{r\cos\theta}{c}\right) dr \quad (10)$$

and

$$G_2(t) = \int_{-\infty}^{\infty} u(r \sin\theta + v_x t) \cdot s\left(t(1 + 2\tfrac{v_z}{c}) - 2\tfrac{r\cos\theta}{c}\right) \cdot u^*((r + \Delta r)\sin\theta + v_x(t+\tau)) \cdot s^*\left((t+\tau)(1 + 2\tfrac{v_z}{c}) - 2\tfrac{(r+\Delta r)\cos\theta}{c}\right) dr. \quad (11)$$

In the special case when the transducer is circularly symmetric, it is straight forward to show that we can always choose the x-axis in our model such that the flow is within the x-z plane. As a result, it is sufficient to obtain the two components on x and z directions to be able to calculate the true velocity magnitude.

Next we consider some special cases of using our model to estimate the velocity of a moving pattern of scatterers.

3. SPECIAL CASE (1): IMPULSE PLANE WAVE EXCITATION

In this theoretical case, the moving scatterers are at the far-field of a very low-Q ideal transducer which was excited by an impulse. That is, $u(x) = 1$ and $s(t) \approx \delta(t)$. Hence, substituting into (10) and (11), we obtain

$$G_1(t) = 1 \quad (12)$$

and

$$G_2(t, \tau) = \delta\left(\tau \cdot (1 + 2\frac{v_z}{c}) - 2\frac{\Delta r\cos\theta}{c}\right). \quad (13)$$

Hence, from the form of (9), it can be shown that the correlation outcome will take the form of a constant plus a single impulse. That is, the peak of the correlation function is given by

$$\tau_{peak} = \frac{2 \cdot \Delta r \cdot \cos\theta}{(1 + 2\frac{v_z}{c}) \cdot c}. \tag{14}$$

Assuming that $v_z \ll c$ and that the velocity was uniform throughout the period of insonification, we obtain the following expression for the velocity:

$$v_z = \frac{\Delta r \cdot \cos\theta}{T} = \frac{\tau_{peak} \cdot c}{2 \cdot T} \tag{15}$$

which is the same expression as the one given in the literature [3]. Notice that it is only possible to measure the axial component of the velocity in this case. As a result, three spatially independent excitations are required to obtain the true-velocity magnitude.

4. SPECIAL CASE (2): BROADBAND PLANE WAVE EXCITATION

In this case, we look at a more practical situation where the transducer has a finite Q-value which translates a spike electrical excitation into a broadband signal centered around its characteristic frequency. That is, we still have $u(x) = 1$ but now $s(t)$ is a general finite energy function of a very small time support. Hence, we obtain

$$G_1(t) = \mathcal{A} = constant \tag{16}$$

and

$$\begin{aligned} G_2(t) \quad &= \int_{-\infty}^{\infty} s\left(t(1 + 2\tfrac{v_z}{c}) - 2\tfrac{r\cos\theta}{c}\right) \\ &\cdot s^*\left((t+\tau)(1 + 2\tfrac{v_z}{c}) - 2\tfrac{(r+\Delta r)\cos\theta}{c}\right) dr. \end{aligned} \tag{17}$$

Notice that $G_2(t, \tau)$ is essentially the autocorrelation function of the scaled and shifted version of the excitation $s(\cdot)$. Then, the correlation function will take the form of a constant plus this autocorrelation function and the peak of the sum is the same as that of the autocorrelation. Under the same assumptions used in Section 3., we obtain the same result. Instead of comparing snapshots of the moving pattern as in Section 3., the comparison here is performed on a smeared versions of these snapshots in this case. This produces a weak smooth peak here instead of the strong peak of the previous case. The peak tends to get even weaker when we increase the length of the excitation signal. It should be noted also that the only possible outcome of this method is a single component of the velocity vector.

5. SPECIAL CASE (3): NARROW-BAND PLANE WAVE EXCITATION

In this case, we consider the case of a transducer with a higher Q-value under the same conditions. So, we still have $u(x) = 1$ and now $s(t) = s'(t) \cdot e^{j\omega_o t}$, where $s'(t)$ is the envelope. Hence, we obtain

$$\begin{aligned} G_1(t) \quad &= e^{j(\omega_o+\omega_z)t} \cdot \int_{-\infty}^{\infty} s'\left(t - \tfrac{2r\cos\theta}{c}\right) dr \\ &\triangleq \mathcal{A} \cdot e^{j(\omega_o+\omega_z)t} \end{aligned} \tag{18}$$

and

$$\begin{aligned} G_2(t) \quad &= e^{-j(\omega_o+\omega_z)\tau} \cdot \int_{-\infty}^{\infty} s'(t - \tfrac{2r\cos\theta}{c}) \cdot \\ &s'^*\left(t + \tau - \tfrac{2(r+\Delta r)\cos\theta}{c}\right) dr \end{aligned} \tag{19}$$

where $\mathcal{A}$ is a constant and ω_z is the classical Doppler shift. Then, the correlation function can be expressed as:

$$\begin{aligned} R_{rr}(t, t+\tau) = e^{-j(\omega_o+\omega_z)\tau} \cdot \Big[\lambda^2 \cdot |\mathcal{A}|^2 \\ + \frac{\lambda c}{2\cos\theta} \cdot \int_{-\infty}^{\infty} s'(\alpha) \cdot s'^*\left(\alpha + \frac{\tau c}{2\cos\theta} - \Delta r\right) d\alpha\Big]. \end{aligned} \tag{20}$$

It can be shown that the envelope of the obtained correlation function is essentially the same as the one obtained in Section 4. with a weaker peak due to the increase in excitation time for narrow-band signals. Also, this envelope is modulated by the original central frequency shifted by the Doppler effect. Hence, a multiple peak pattern is expected for this case and the accuracy of determining the exact location of the peak is impaired by the fact that the modulating signal has to have one of its peaks exactly on top of the autocorrelation peak to appear as the single global peak. This might suggest a demodulation step before performing the correlation calculations.

6. SPECIAL CASE (4): NARROW-BAND FOCUSSED BEAM EXCITATION

We now consider the practical situation of flow mapping where we are concerned also with the problem of resolution in both the axial and lateral directions. In this case, $u(x)$ is typically a Gaussian-like function and $s(t) = s'(t) \cdot e^{j\omega_o t}$ is essentially a number of cycles of a central frequency ω_o with a slowly varying envelope $s'(t)$. Hence,

$$\begin{aligned} G_1(t) \quad &= e^{-j(\omega_o+\omega_z)t} \int_{-\infty}^{\infty} u(v_x t + r\sin\theta) \cdot s'\left(t - 2\tfrac{r\cos\theta}{c}\right) dr \\ &\triangleq \mathcal{A}(t) \cdot e^{-j(\omega_o+\omega_z)t} \end{aligned} \tag{21}$$

and

$$\begin{aligned} G_2(t) = e^{-j(\omega_o+\omega_z)\tau} \cdot \int_{-\infty}^{\infty} u\left(t + \tfrac{r\sin\theta}{v_x}\right) \cdot s'\left(t - 2\tfrac{r\cos\theta}{c}\right) \\ \cdot u'^*\left(t + \tau + \tfrac{(r+\Delta r)\sin\theta}{v_x}\right) \cdot s'^*\left((t+\tau) - 2\tfrac{(r+\Delta r)\cos\theta}{c}\right) dr \\ \triangleq \mathcal{B}(t,\tau) \cdot e^{-j(\omega_o+\omega_z)\tau}. \end{aligned} \tag{22}$$

Then, the correlation function takes the form:

$$R_{rr}(t, t+\tau) = e^{-j(\omega_o+\omega_z)\tau}\left[\lambda^2 A(t) A^*(t+\tau) + \lambda B(t,\tau)\right]. \tag{23}$$

A number of important observations can be drawn from this case. As we can see, (23) is a function of both $\cos\theta$ and $\sin\theta$. This indicates that in general both velocity components affect the outcome of the technique unlike the previous cases. It is clear also that the outcome will depend on the relative characteristics of both the envelope of the excitation signal and the ultrasound field. For example, if the support of $u(\cdot)$ is sufficiently larger than that of $s'(\cdot)$, the result will depend only on the $\cos\theta$ part in very much the same way as the above cases. On the other hand, if $s'(\cdot)$ is sufficiently long, the outcome will depend only on the characteristics of the field profile $u(\cdot)$ which is a direct function of the lateral velocity v_x. It can be shown that the correlation peak location will be fixed at $-T$. This means that the correlation peak can only be obtained from an autocorrelation measure. Also, the width W of the correlation function will be a function of the lateral velocity v_x. In particular, we have

$$v_x = \frac{\mathcal{D}}{W}. \tag{24}$$

Here W is the calculated time domain correlation width, and $\mathcal{D}$ is a constant with units of distance and is defined as the spatial width of the beam for a unity v_x. The definition of the width here is not unique as long as $\mathcal{D}$ is modified accordingly. In this case, we are essentially tracking the lateral motion of the scattering pattern while smearing out the effect of the axial motion.

7. SPECIAL CASE (5): BROADBAND FOCUSSED BEAM EXCITATION

In this case, the excitation signal is usually very short such that the effect of the ultrasound field in determining the peak is negligible. In theory, the beam pattern should have a very small bias toward the spatial correlation peak and thus tends to give slightly lower values for the peak location. So, the location of the peak will be practically determined by the axial velocity component. This is easy to realize by looking into the expressions for the correlation times of of the lateral and axial correlations which have differ by the ratio of v_x to c, which might be larger than 1000. Typical values for τ_{peak} are hundreds of nanoseconds while T values may be more than $100\mu s$. Nevertheless, as time goes on, decorrelation results from the effect of moving parts of the scattering pattern away from the ultrasound field, and this will affect the correlation peak magnitude [3].

8. LATERAL VELOCITY ESTIMATION VIA DECORRELATION TRACKING

In the simple case of a rectangular field pattern, the theory suggests that by measuring several correlation peaks instead of only one, we can estimate the ultrasound field width by extrapolation. This width is a direct function of the lateral velocity as in (24). In other words, tracking the decorrelation in the return signal due to the lateral motion of the scattering pattern can indeed be used to estimate the lateral velocity in this simple case. Moreover, it can be shown that in the case of any field pattern, the obtained fit will give a valid bandwidth measure that can be still be used to estimate the lateral velocity. Therefore, the result is general. The idea of the technique is shown is Figs.(2) and (3).

In the practical situations where noise is likely to affect the measured peaks, a least square fit may be used to obtain better results. Hence, it is possible to obtain both velocity components from the time domain correlation using at least 3 excitations.

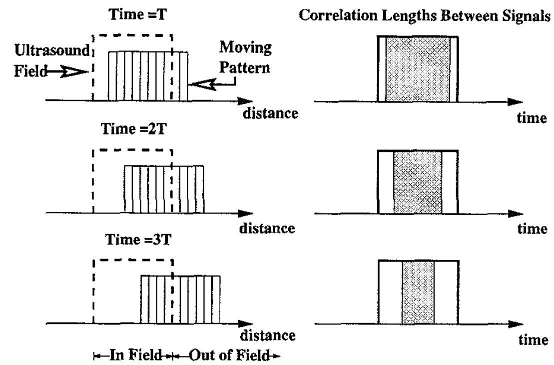

Fig.(2) Decorrelation effect

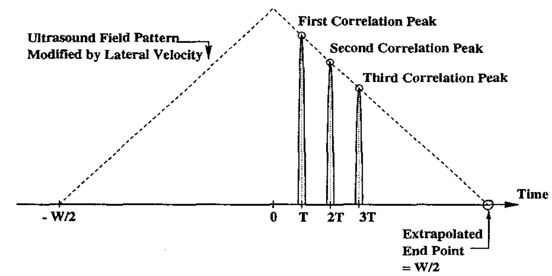

Fig.(3) Lateral velocity estimation

9. CONCLUSIONS

The axial velocity estimation using the classical formula of the time domain correlation technique from a demodulated signal is not likely to be affected by the beam pattern under normal imaging conditions. However, a decorrelation results from the beam shape while the random pattern is moving away from the field which affects the correlation peak. This decorrelation can be measured and extrapolated to obtain valuable information about the effective beam pattern width, which is a direct function of the lateral velocity. Given these measurements from a circularly symmetric aperture, it is possible to reconstruct the magnitude of the 3-d velocity vector thus enabling to create true-velocity maps using the correlation technique.

10. REFERENCES

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