MEASUREMENT

EE 306 – SS2016

Prof. Yasser Mostafa Kadah – www.k-space.org

Measurement Basics

- Measuring is the experimental determination of a measured value by quantitative comparison of the measurand with a comparison value in a direct or indirect manner
- Measured value obtained by this procedure is given as a product of a numeric value and a dimensional unit
- It can be recorded continuously as a temporal variation of a physical value or discontinuously at particular moments
- Deviation of measured value from the measurand is the measurement error
  - Depends on measurement procedure, measurement device, and environmental effects
  - Systematic and random errors are distinguished

Gross (Human) Errors

- Reading the instrument before it has reached its steady state.
- Parallax error when reading an analog meter scale.
- Mistakes in recording measured data and in calculating a derived measurand
- Misuse of the instrument

Viewing from the left

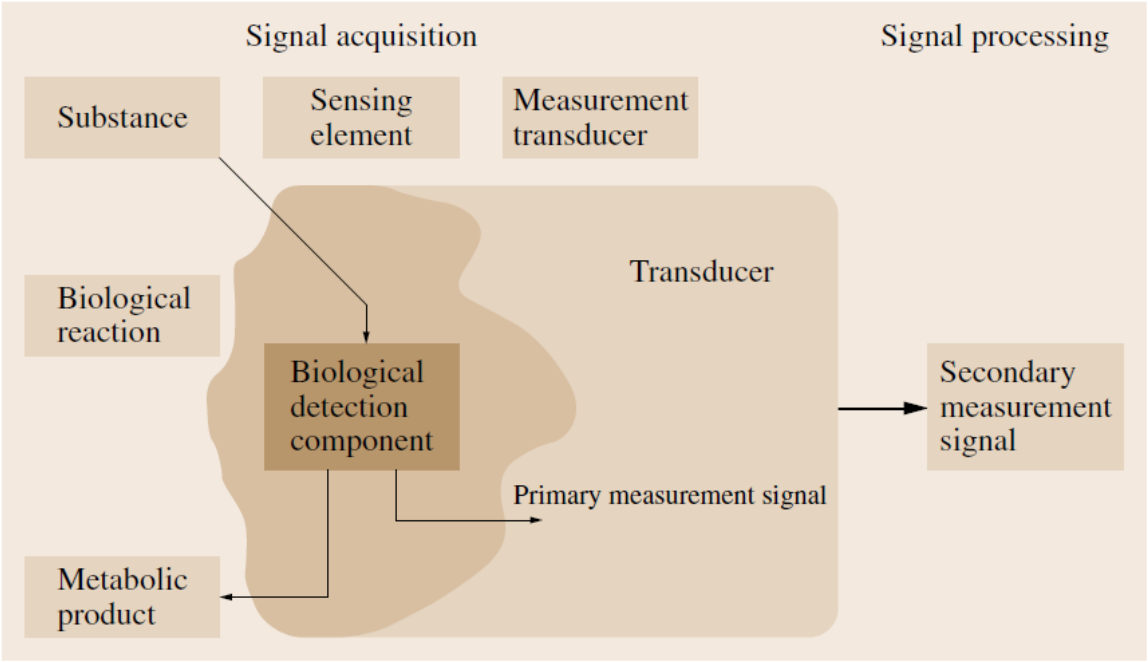
Viewing from the right

Systematic Error

- Method of measurement has an error
- Instrument is not calibrated or has an offset
  - Loss of calibration and zero error can occur because of long term component value changes due to aging, or changes associated with temperature rise
- Reading uncertainty due to presence of random noise
  - External noise from *environmental noise* can be reduced by appropriate electric and magnetic shielding/grounding
  - Internal noise (e.g., from an instrument's signal conditioning)
- Reading uncertainty due to slow, or long term drift in the system
  - Drifts can cause slow changes in system sensitivity and/or zero. Drift may arise as the result of a slow temperature change as a system warms up. Drift or system offset can also arise from dc static charges.

Sensor

- Sensor is a probe to register measured events
- Often, it is directly connected to a transducer, or it transduces the primary measurement signal into a secondary signal itself



Ideal Sensor Requirements

- Feedback-free registration of the signals
- Provide reproducible measurement results
- Transmission behavior has to remain constant for a long time
- Narrow production tolerances
- Small mass and small volume
- Application should be simple and manageable

- (BME) High biocompatibility
- (BME) Low stress to patient
- (BME) Allow cleaning, disinfection and possibly sterilization

Sensor Classification

- Active or passive
- Passive sensor does not need any additional energy source
  - Directly generates electric signal in response to external stimulus
  - Examples: thermocouple, photodiode, piezoelectric transducer
- Active sensors require external power for their operation, called excitation signal.
  - Excitation signal is modified by sensor to produce the output signal
  - Examples: thermistor and resistive strain gauge

Sensor Classification

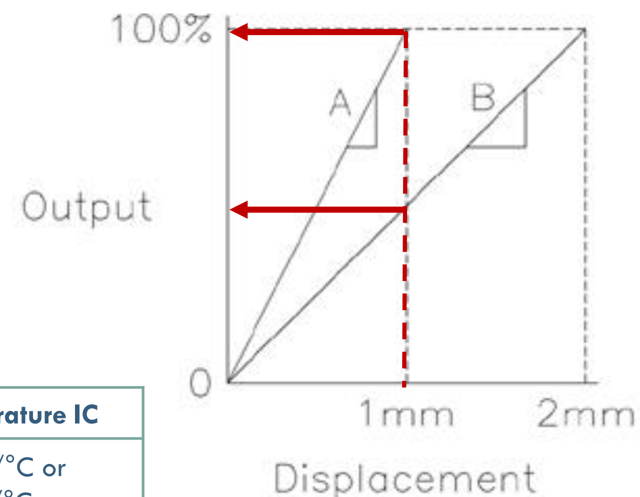
- Absolute or relative
- Absolute sensor detects a stimulus in reference to an absolute physical scale that is independent of the measurement conditions
  - Example: thermistor – electrical resistance directly related to absolute temperature scale of Kelvin
- Relative sensor produces a signal that relates to some special case
  - Example: thermocouple – produces electric voltage that is function of temperature gradient across the thermocouple wires

Sensor Specifications

Sensitivity	Stimulus range (span)
Stability (short and long term)	Resolution
Accuracy	Selectivity
Speed of response	Environmental conditions
Overload characteristics	Linearity
Hysteresis	Dead band
Operating life	Output format
Cost, size, weight	Other

Sensor Sensitivity

- Sensitivity is typically defined as the ratio of output change for a given change in input
  - Another definition can be given as the slope of the calibration line relating the input to the output (i.e., ΔOutput/ΔInput)
- Example: Sensor A is more sensitive than sensor B
  - Same displacement, higher output from A
- Example: Temperature sensors



Characteristic	Platinum RTD	Thermistor	Thermocouple	Temperature IC
Sensitivity	2 mv/°C	40 mV/°C	0.05 mV/°C	~1 mv/°C or ~1 uA/°C

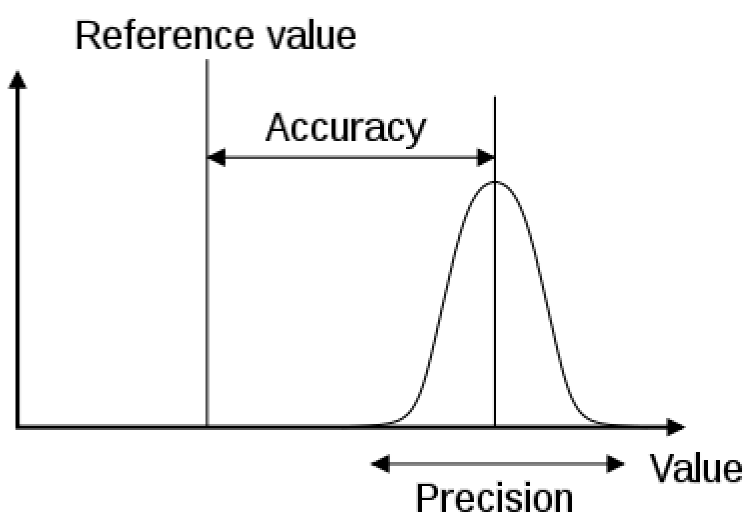
Sensor Dynamic Range

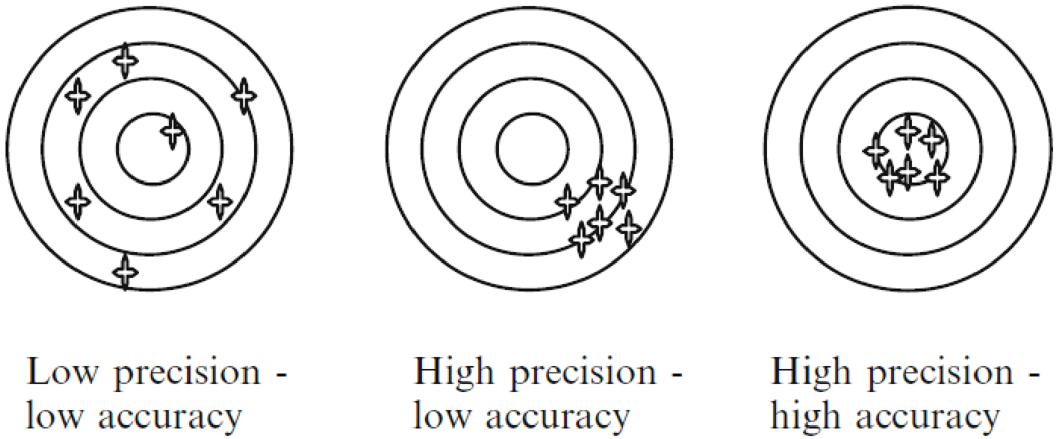
- Dynamic range of a sensor corresponds to the minimum and maximum operating limits that the sensor is expected to measure accurately
  - Also called stimulus range or span
- Example: Temperature sensors have very different ranges that suit different applications
  - From measuring human temperature to measuring temperature in steam sterilizers

Characteristic	Platinum RTD	Thermistor	Thermocouple	Temperature IC
Temperature Range	-200°C to 500°C	-40°C to 260°C	-270°C to 1750°C	-55°C to 150°C

Sensor Accuracy and Precision

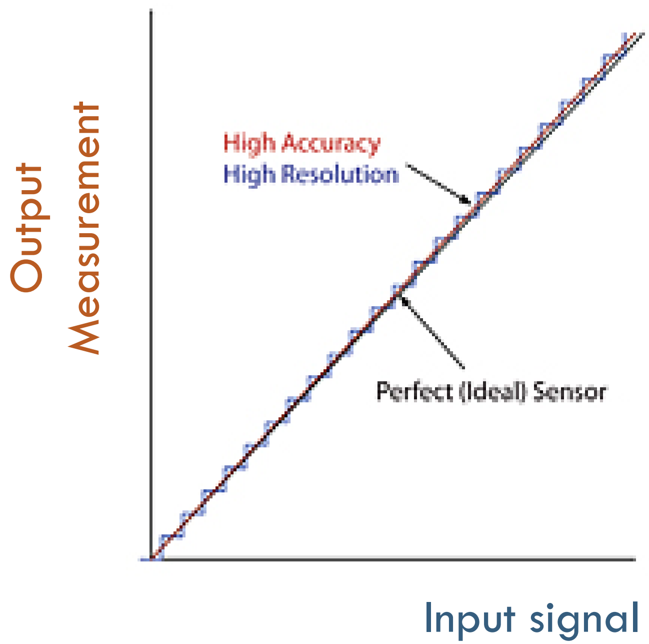
- Accuracy refers to the difference between the true value and the actual value measured by the sensor
- Precision refers to degree of measurement reproducibility
  - Very reproducible readings indicate a high precision
- Precision should not be confused with accuracy
  - Measurements may be highly precise but not necessary accurate

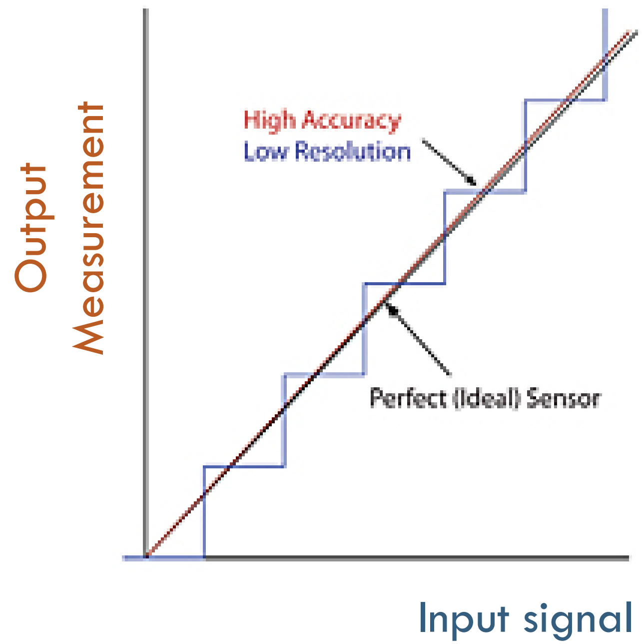




Sensor Resolution

- Resolution is defined as the smallest change of the measurand that can produce a detectable change in the output signal
- Example: sensors with digital output only change in steps of 1 bit
  - 12-bit sensors will have better resolution than 8-bit sensors



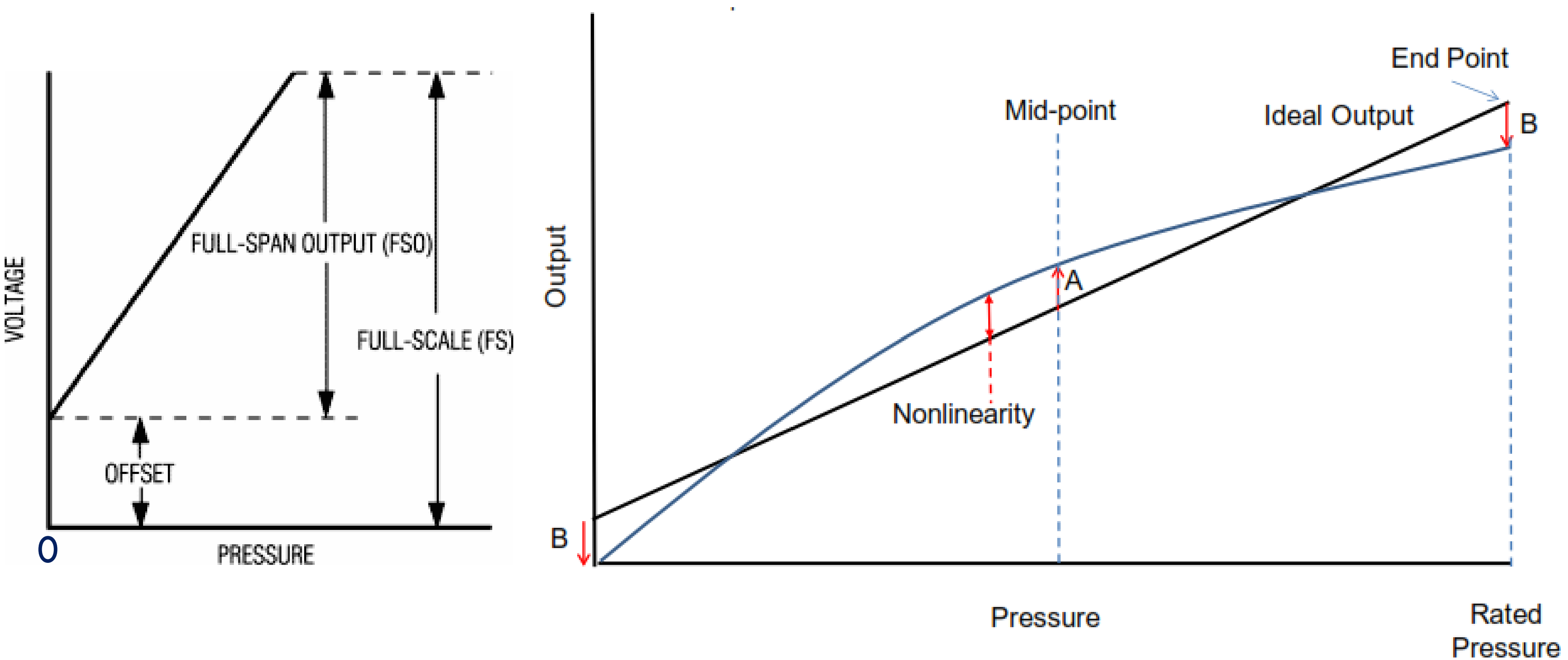


Sensor Reproducibility

- Reproducibility is the degree to which an experiment or study can be accurately reproduced, or replicated, by someone else working independently or over time
  - Sometimes called repeatability or stability (short-term and long-term)
- Reproducibility can vary depending on the measurement range
  - Readings may be highly reproducible over one range and less reproducible over a different operating range

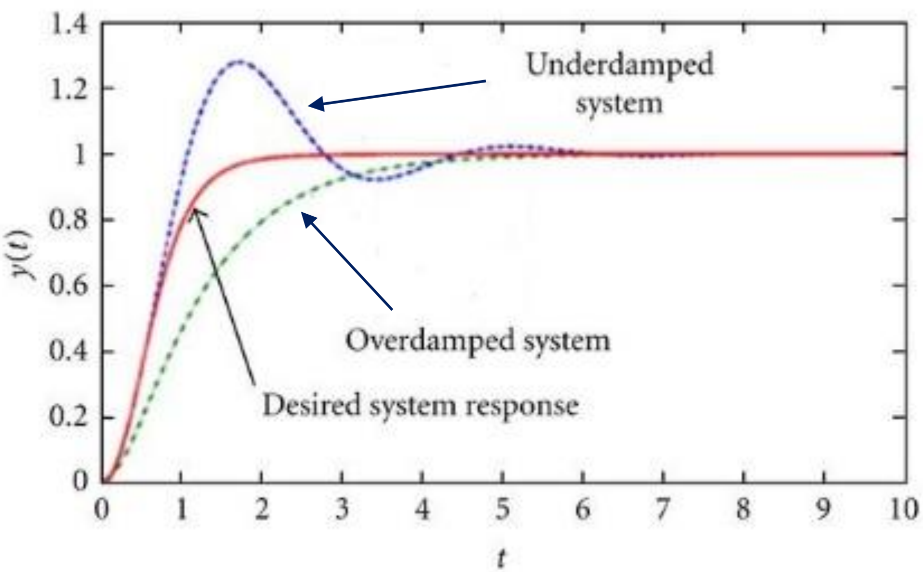
Sensor Linearity and Offset

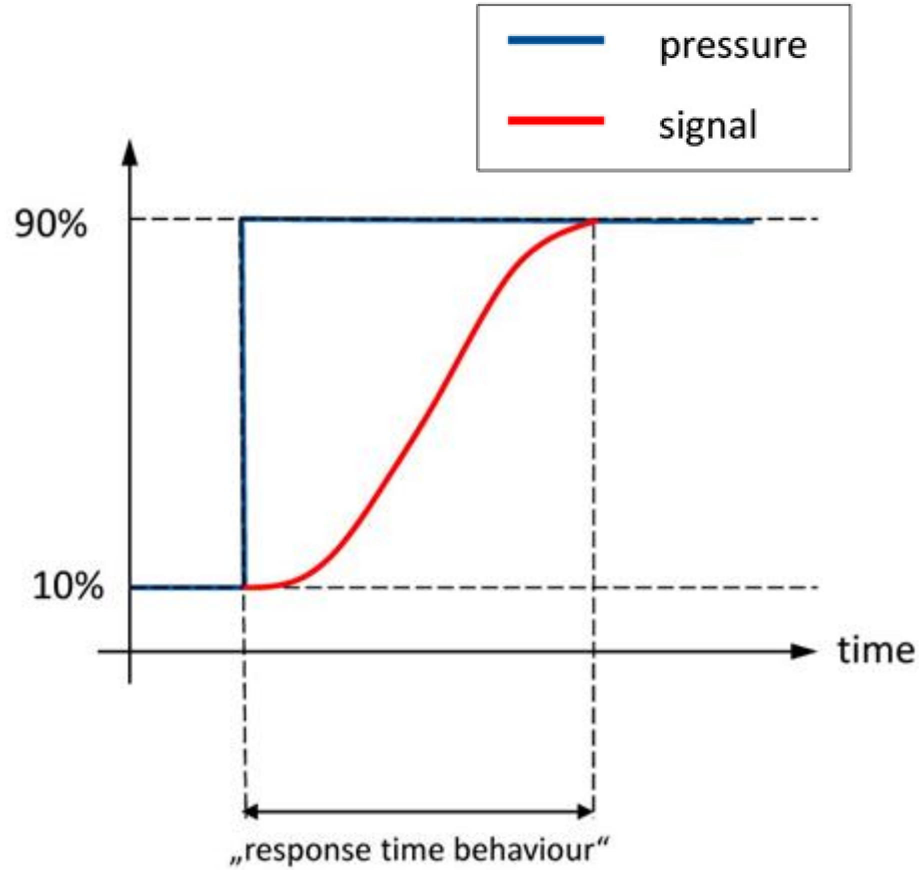
- Linearity is a measure of the maximum deviation of any reading from a straight calibration line
- Offset refers to the output value when the input is zero



Sensor Response Time

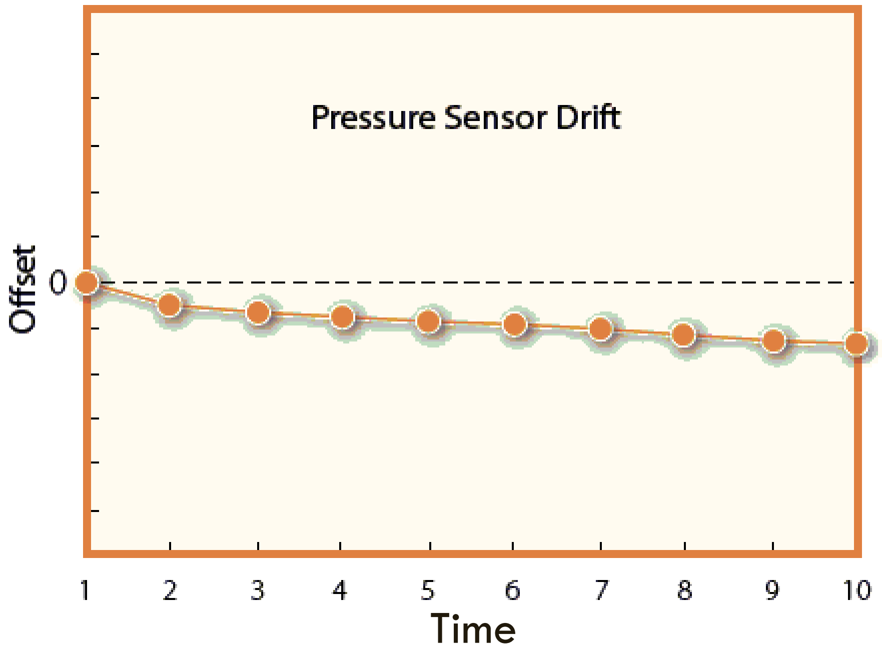
- Response time indicates the time it takes a sensor to reach a stable (steady-state) value when the input is changed
  - Same as recovery time

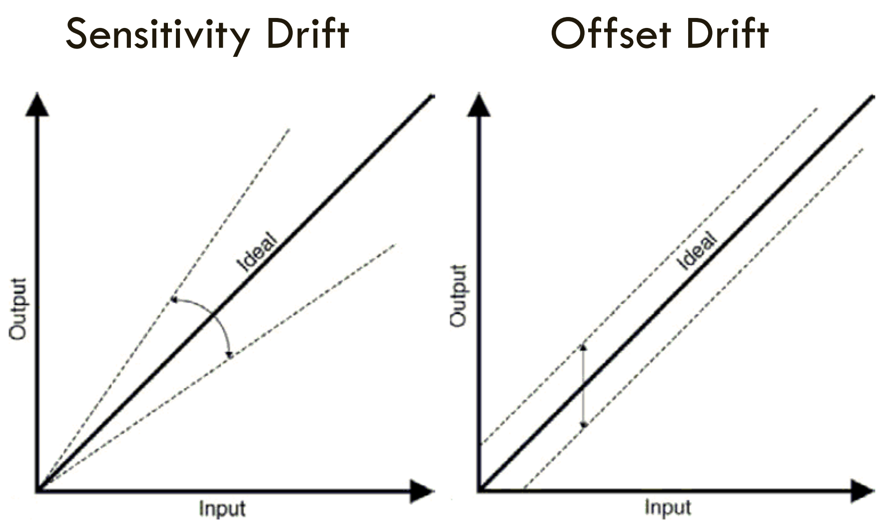




Sensor Drift

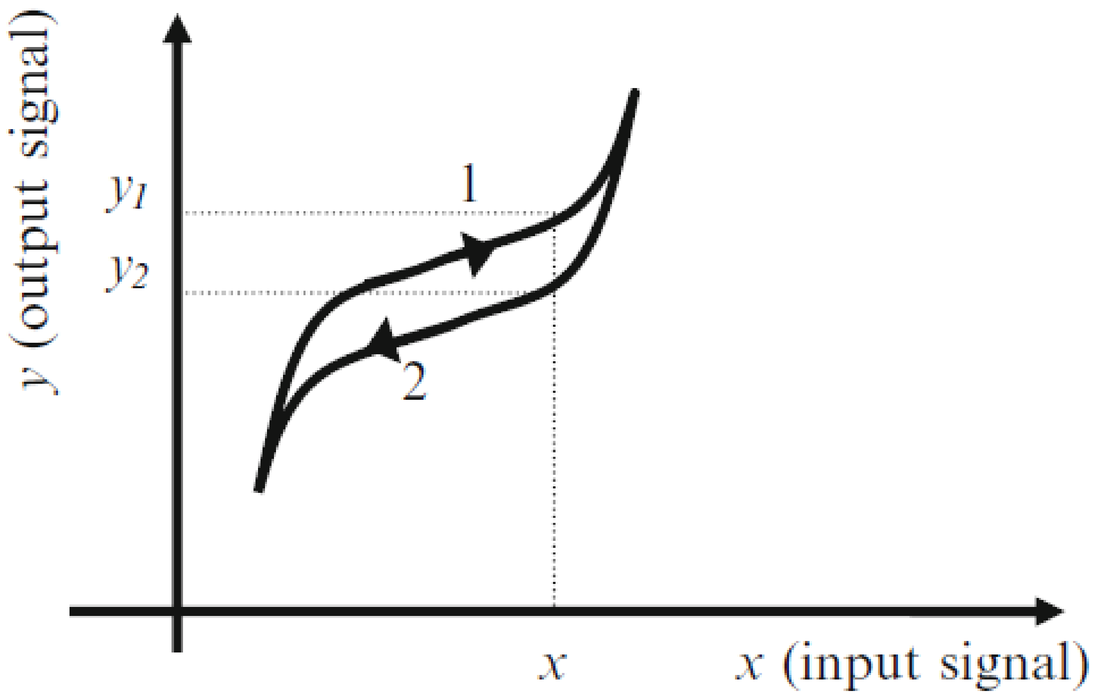
- Drift is a gradual change in the measurement output is seen while the measurand actually remains constant
  - Drift is undesired systematic error that is unrelated to the measurand
  - Drift may affect offset and/or sensitivity





Sensor Hysteresis

- Hysteresis is the difference between output readings for the same measurand, depending on the trajectory followed by the sensor
  - Depending on whether path 1 or 2 is taken, two different outputs are obtained for the same input



Mathematical Definition of Error

- Error: deviation of the measured value from the true value.
- True value: standard or reference of known value or theoretical value
- Error in n^th measurement is given as

$$\varepsilon_n = X_n - Y_n$$

$$\% \, \varepsilon = |\varepsilon_n / Y_n| \times 100\%$$

  - $X_n = \text{n}^{\text{th}}$ measured value
  - $Y_n =$ actual, true, defined or calculated value

Limiting Error

- Limiting Error (LE) is an important parameter used in specifying instrument accuracy given by manufacturers to define the outer bounds or the expected worst case error
  - For example, when a voltmeter specified as having accuracy of 2% of its full-scale reading on the 100 V scale has reading of 75 V, LE in this reading is $(2/75) \times 100 = 2.67\%$

Measurement Uncertainty Analysis

- In many cases, measurement must be calculated from a formula with various system parameters, each having a specified accuracy
  - Need to derive formula for LE in value of measurement
- Let measurement be function of N variables as,

$$Q = f\left(X_1, X_2, \ldots, X_N\right)$$

- Assuming each variable, X_i, has error $\pm\Delta X_i$, then calculated measurement will contain error and will be given by,

$$\hat{Q} = f\left(X_1 \pm \Delta X_1, X_2 \pm \Delta X_2, \ldots, X_N \pm \Delta X_N\right)$$

- Idea: Use Taylor series expansion to get an approximate expression

Measurement Uncertainty Analysis

- Taylor series expansion is given as,

$$f(X \pm \Delta X) = f(X) + \frac{\mathrm{d}f}{\mathrm{d}X}\frac{\Delta X}{1!} + \frac{\mathrm{d}^2 f}{\mathrm{d}X^2}\frac{(\Delta X)^2}{2!} + \cdots + \frac{\mathrm{d}^{n-1} f}{\mathrm{d}X^{n-1}}\frac{(\Delta X)^{n-1}}{(n-1)!} + R_n$$

- Expanding $\hat{Q}$ using the above expansion,

$$\hat{Q} = f(X_1, X_2, \ldots, X_N) + \left\{ \frac{\partial f}{\partial X_1}\Delta X_1 + \frac{\partial f}{\partial X_2}\Delta X_2 + \cdots + \frac{\partial f}{\partial X_N}\Delta X_N \right\}$$

$$+ \frac{1}{2!}\left\{ \frac{\partial^2 f}{\partial X_1^2}(\Delta X_1)^2 + \frac{\partial^2 f}{\partial X_2^2}(\Delta X_2)^2 + \cdots + \frac{\partial^2 f}{\partial X_N^2}(\Delta X_N)^2 \right\} + \cdots$$

$$+ \frac{1}{3!}\left\{ \frac{\partial^3 f}{\partial X_1^3}(\Delta X_1)^3 + \cdots \right\} + \cdots.$$

$\cong 0$

Second and higher derivative terms can usually be assumed to be numerically negligible

Measurement Uncertainty Analysis

- Hence, the reading containing error can be approximated as:

$$\hat{Q} \cong f(X_1, X_2, \ldots, X_N) + \left\{ \frac{\partial f}{\partial X_1} \Delta X_1 + \frac{\partial f}{\partial X_2} \Delta X_2 + \cdots + \frac{\partial f}{\partial X_N} \Delta X_N \right\}$$

True Measurement

Measurement Error

- Maximum or worst-case uncertainty in Q can be approximated by,

$$\Delta Q_{MAX} = \left| Q - \hat{Q} \right| = \sum_{j=1}^{N} \left| \frac{\partial f}{\partial X_j} \Delta X_j \right|$$

Measurement Uncertainty Analysis: Example

- Derive LE in calculation of DC power in resistor

$$Q = f(X_1, X_2, \ldots, X_N) \quad \Rightarrow \quad P = I^2 R$$

$$\Delta Q_{\mathrm{MAX}} = \left|Q - \hat{Q}\right| = \sum_{j=1}^{N} \left| \frac{\partial f}{\partial X_j} \Delta X_j \right| \quad \Rightarrow \quad \Delta P_{\mathrm{MAX}} = 2IR\Delta I + I^2 \Delta R$$

$$\frac{\Delta P_{\mathrm{MAX}}}{P} = 2\left|\frac{\Delta I}{I}\right| + \left|\frac{\Delta R}{R}\right|$$

- If LE in *R* is 0.1%, 0–10 A ammeter has 1% of full-scale accuracy, resistor value is 100 Ω and ammeter reads 8 A, and nominal power dissipated in resistor is 6400 W, then LE in power measurement is:

$$\frac{\Delta P_{\mathrm{MAX}}}{P} = 2\frac{0.1}{8} + 0.001 = 0.026 \quad \text{or} \quad 2.6\%$$

Further Reading and Assignments

- Hughes Chapter 44.2, 46.16
- Northrop Chapter 1.3
- Chapter 46 of *Springer Handbook of Medical Technology (2011)*
- Chapter 4 of *Sensors in Biomedical Applications* (2000)
- Chapter 2 of Sensors, An Introductory Course (2013)