CAPACITANCE AND CAPACITORS

EE 306 – SS2015

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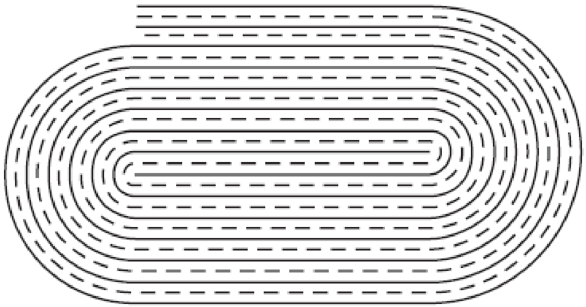
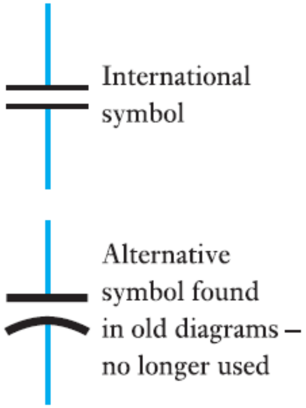
Basics

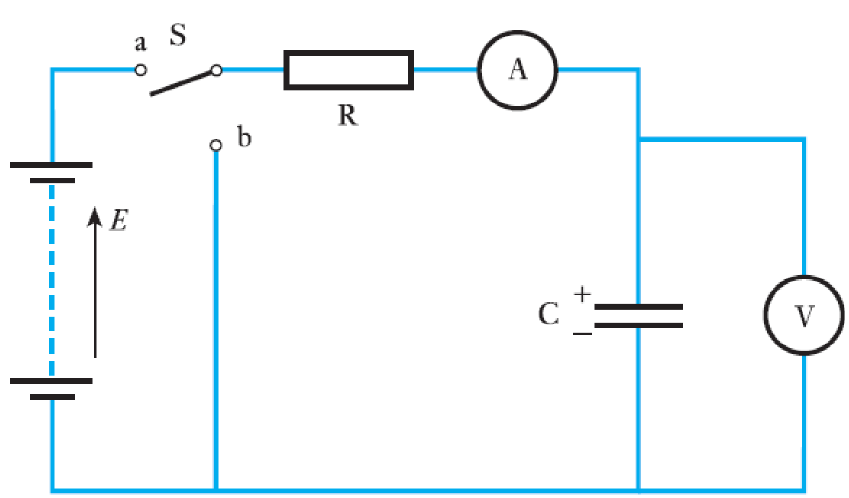
- Capacitor or Condenser is a device which can store electric charge for short periods of time
  - Just like filling container with water, it takes time to pour charge into capacitor
- Conductors would hold much greater electric charges provided that they were held in close proximity to one another yet kept apart
  - The greater surface area of conductors the greater stored charge
  - Simple capacitor can be made from two strips of metal foil sandwiched with two thin layers of insulation (e.g., paper)

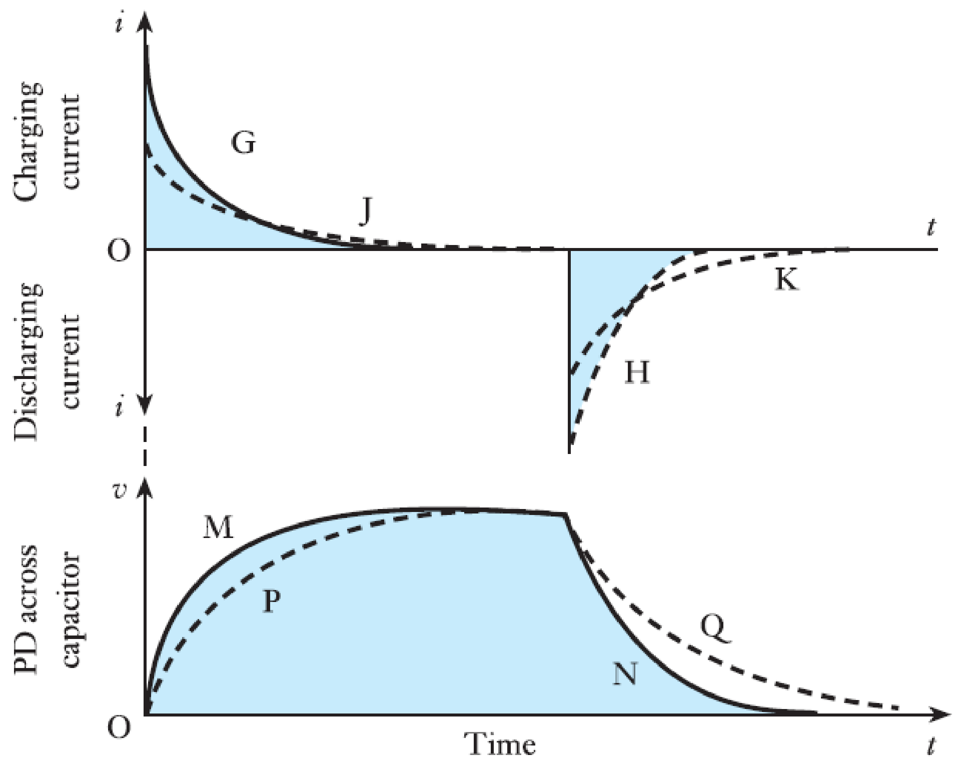




Basics

- Capacitor's ability to hold electric charge is measured in farads
  - Very large unit and most capacitors are rated in microfarads or less
- Charged capacitor may be regarded as reservoir of electricity that can be charged/discharges





Charge and Voltage

- For a given capacitor,

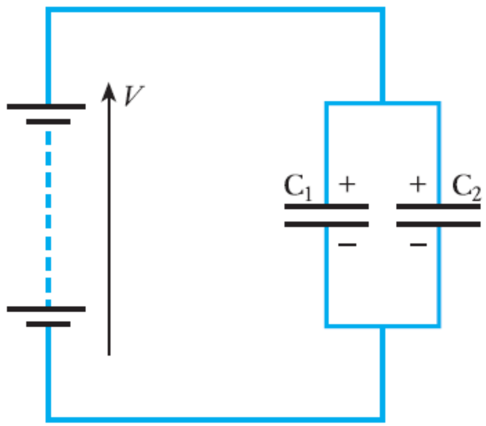
$$\frac{\text{Charge on C [coulombs]}}{\text{PD across C [volts]}} = \text{a constant} = \text{capacitance [farads]}$$

- The property of capacitor to store electric charge when its plates are at different potentials is referred to as its *capacitance*
  - Unit of capacitance is termed the *farad* (F) defined as *capacitance of capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by 1 coulomb of electricity*

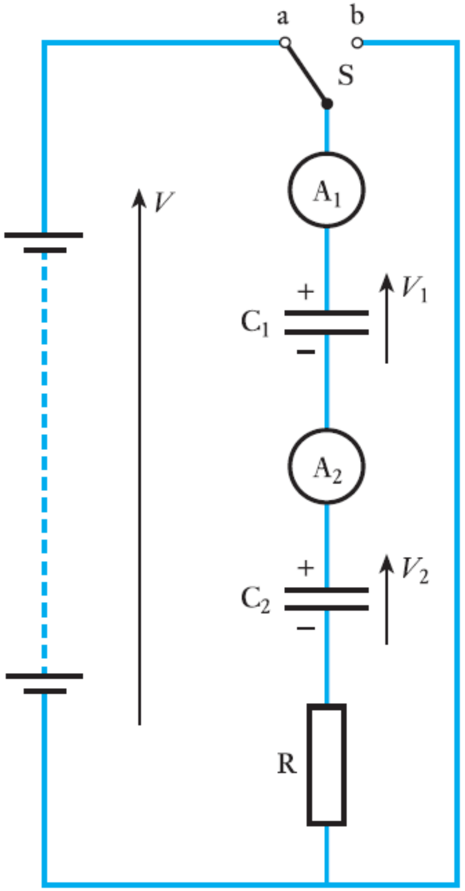
Capacitance Symbol: C Unit: **farad (F)**

$$Q = CV \quad \text{coulombs}$$

Capacitors in Parallel and in Series



$$C = C_1 + C_2$$

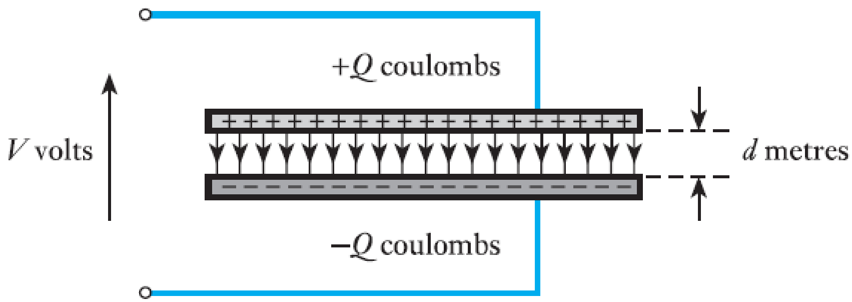


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{V_2}{V_1} = \frac{C_1}{C_2}$$

$$V_1 = V \times \frac{C_2}{C_1 + C_2}$$

Parallel Plate Capacitor

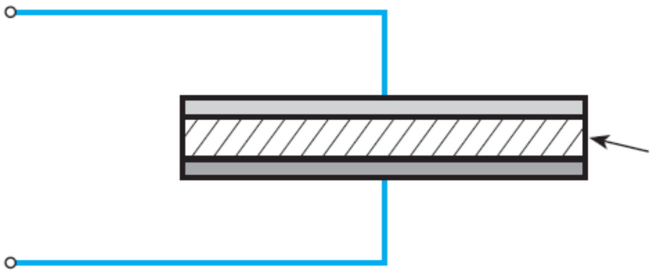


$$C = \frac{\epsilon_0 A}{d} \quad \text{farads}$$

Permittivity of free space Symbol: ϵ_0 Unit: farad per metre (F/m)

8.85×10^{-12} F/m

Parallel Plate Capacitors – General Case



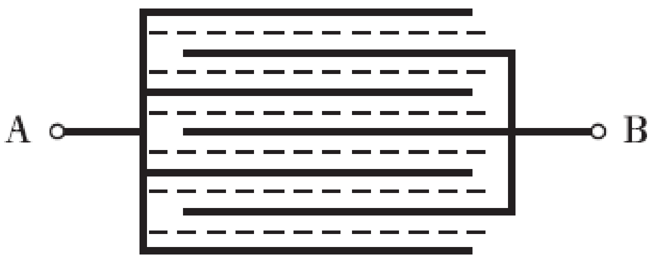
Material	Relative permittivity
Vacuum	1.0
Air	1.0006
Paper (dry)	2–2.5
Polythene	2–2.5
Insulating oil	3–4
Bakelite	4.5–5.5
Glass	5–10
Rubber	2–3.5
Mica	3–7
Porcelain	6–7
Distilled water	80
Barium titanate	6000+

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \quad \text{farads}$$

Relative permittivity Symbol: ϵ_r Unit: none

Capacitance of Multi-Plate Capacitor

- Suppose a capacitor to be made up of n parallel plates



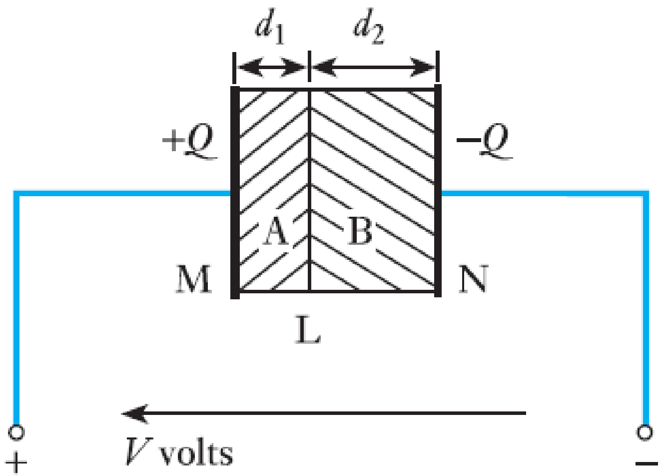
A = area of *one* side of each plate in square metres

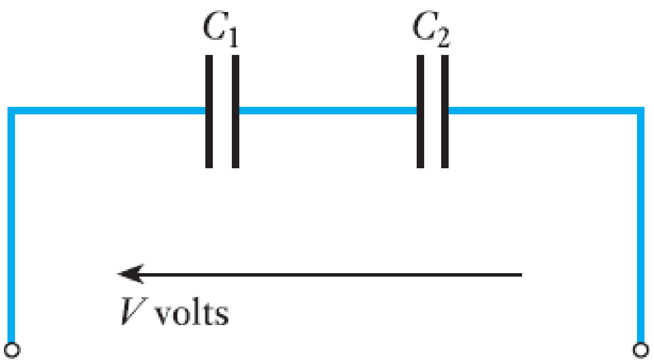
d = thickness of dielectric in metres

ϵ_r = relative permittivity of the dielectric

$$\text{Capacitance} = \frac{\epsilon_0 \epsilon_r (n-1) A}{d} \quad \text{farads}$$

Composite Dielectric Capacitor





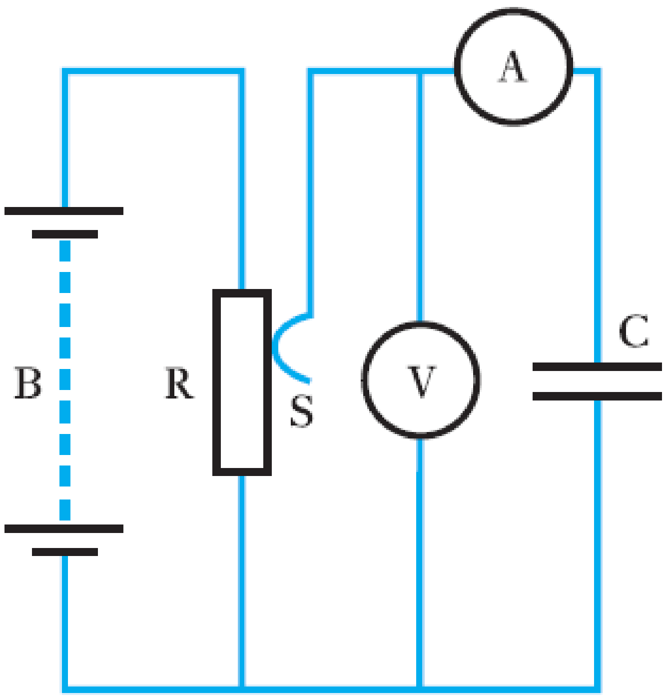
$$C_1 = \frac{\epsilon_1 \epsilon_0 A}{d_1} \quad \text{and} \quad C_2 = \frac{\epsilon_2 \epsilon_0 A}{d_2}$$

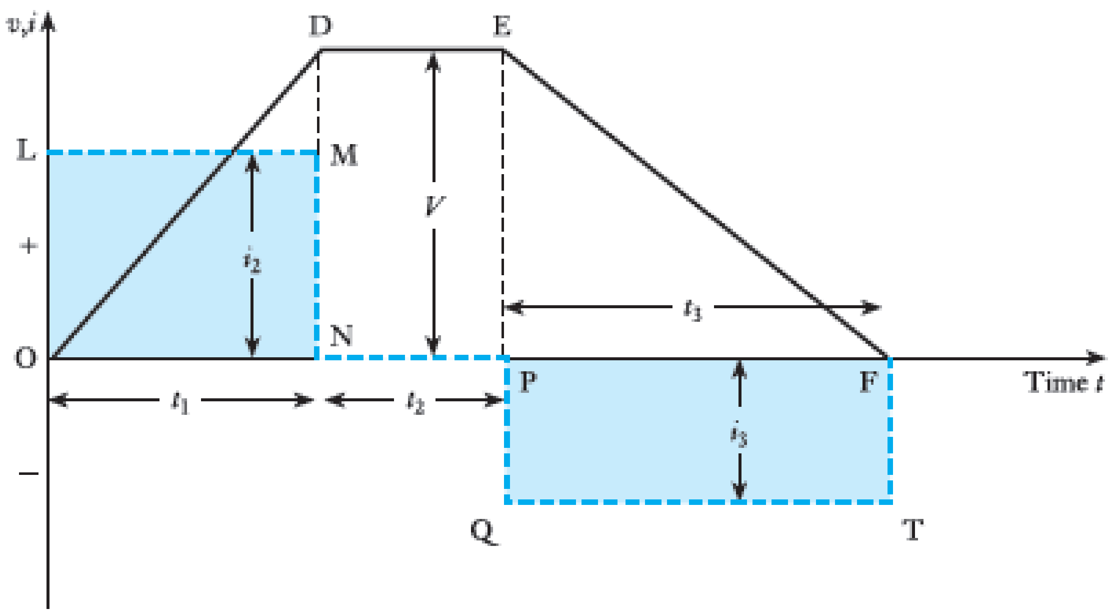
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Charging and Discharging Currents

$$i = \frac{dq}{dt}$$

$$i = C \cdot \frac{dv}{dt}$$



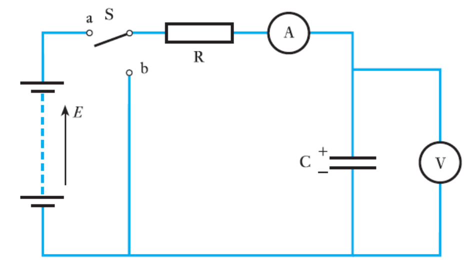


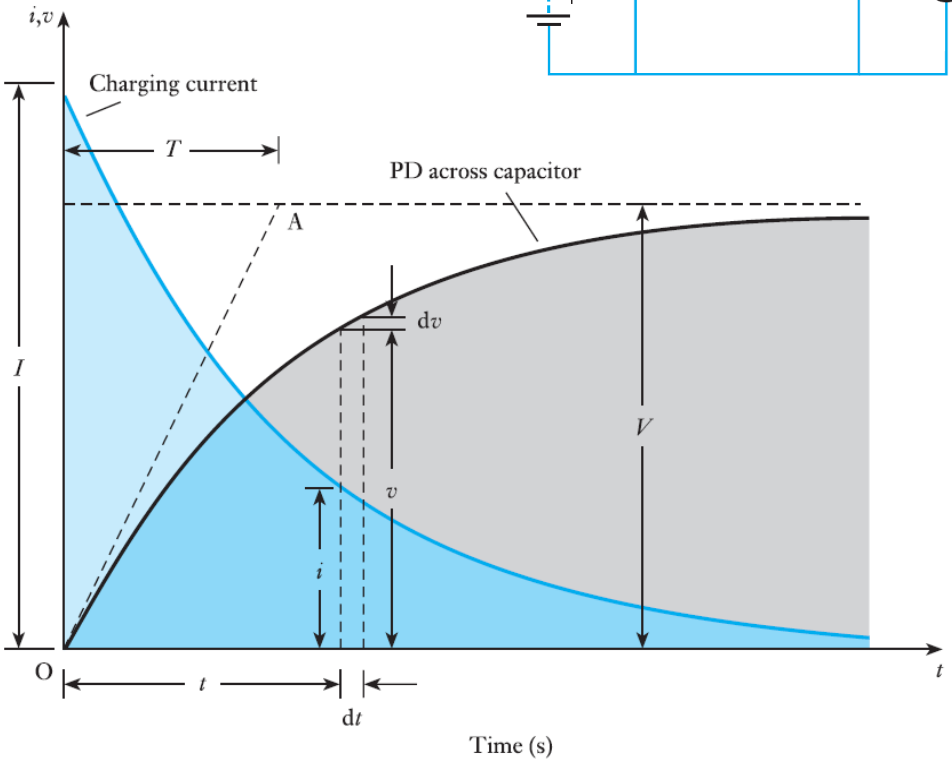
Exponential Growth and Decay

$$v = V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$T = CR$$





Time constant Symbol: T Unit: second (s)

Energy Stored in Charged Capacitor

- Instantaneous value of power to capacitor is,

$$iv \text{ watts} = vC \cdot \frac{dv}{dt} \text{ watts}$$

- Total energy can be given as,

$$\int_0^V Cv \cdot dv = \tfrac{1}{2}C\left[v^2\right]_0^V = \tfrac{1}{2}CV^2 \text{ joules}$$

$$W = \tfrac{1}{2}CV^2$$

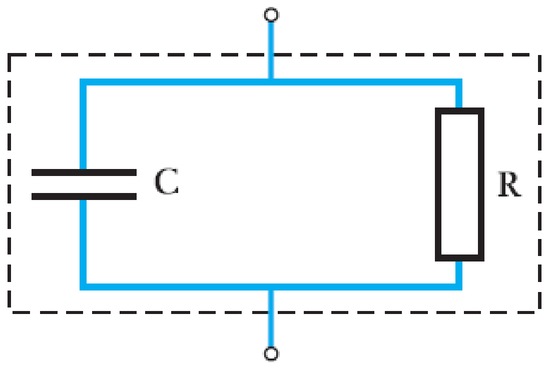
Dielectric Strength

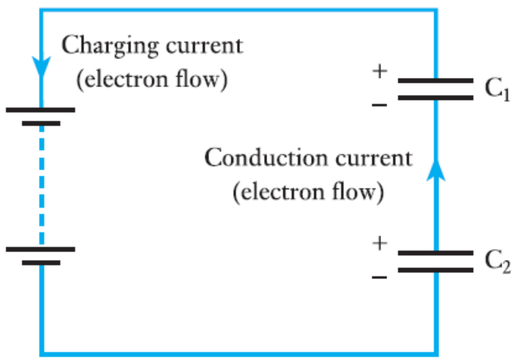
- If the p.d. between the opposite sides of sheet of solid insulating material is increased beyond certain value, material breaks down
- Potential gradient necessary to cause breakdown of insulating medium is termed its dielectric strength (megavolts per meter)

Material	Thickness (mm)	Dielectric strength (MV/m)
Air (at normal pressure and temperature)	0.2	5.75
	0.6	4.92
	1	4.46
	6	3.27
	10	2.98
Mica	0.01	200
	0.1	176
	1.0	61
Glass (density 2.5)	1	28.5
	5	18.3
Ebonite	1	50
Paraffin-waxed paper	0.1	40–60
Transformer oil	1	200
Ceramics	1	50

Leakage and Conduction Currents in Capacitors

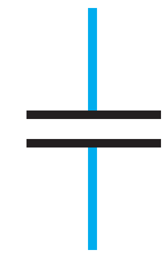
- Every dielectric has few free electrons and therefore effectively acts as insulator of very high resistance between plates of capacitor
  - Practical capacitor has equivalent circuit of parallel R-C
  - Usually, resistance has value in excess of 100 MΩ
  - When voltage is applied across capacitor plates, small *leakage current* passes between plates – charge decays with time
- Leakage is different from *conduction current*
  - Flow of electrons which does not pass through the battery



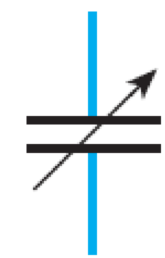


Types of Capacitors

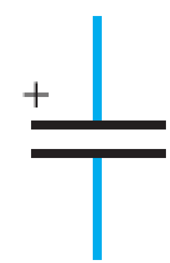
- Fixed capacitors
  - Paper – electrolytic – ceramic – mica – tantalum
- Variable capacitors



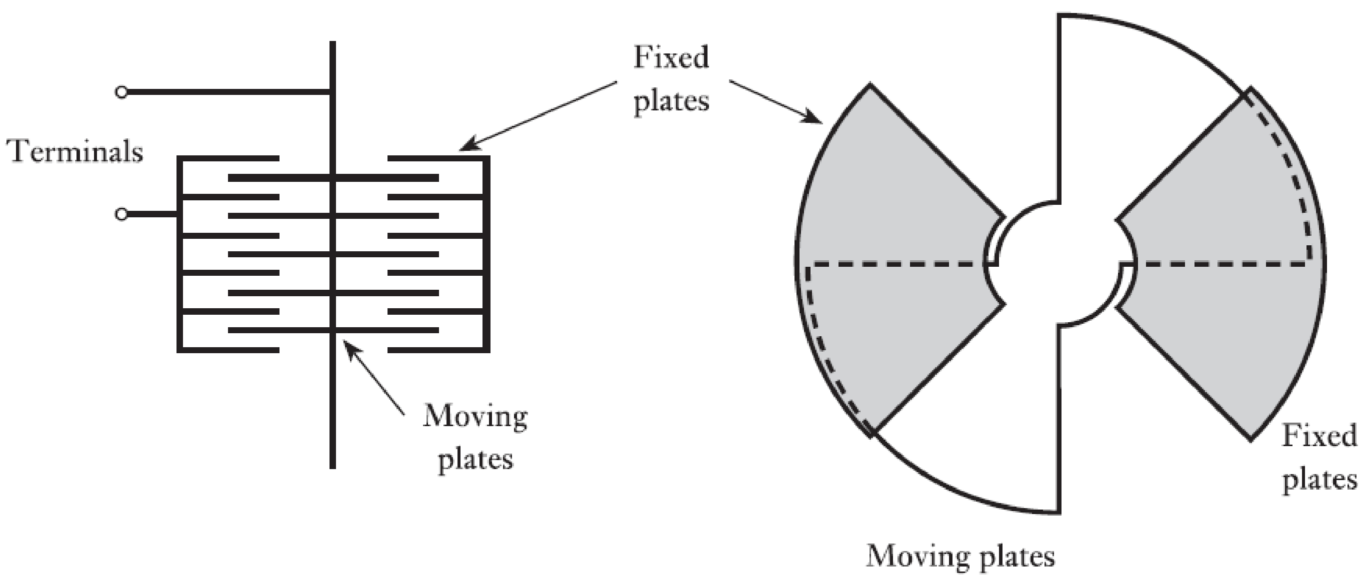
Fixed capacitor



Variable capacitor



Electrolytic capacitor



Summary

Capacitance is a measure of the ability to store electric charge.

Capacitance is also a measure of the ability to store energy in an electric field.

Charging is the process of increasing the charge held in a capacitor.

Discharging is the process of reducing the charge held in a capacitor.

Farad is the capacitance of a capacitor which has a p.d. of 1 V when maintaining a charge of 1 C.

Leakage current is the rate of movement of charge through a dielectric.

Permittivity is the ratio of electric flux density to electric field strength measured in farads per metre.

Suggested Readings and Exercises

- Hughes textbook – Chapters 5
- Exercise 5 (Hughes)
  - Problems 1, 4, 5, 19, 20, 23, 34, 35, 40