BIOINSTRUMENTATION PHYSICS

EE 370 – 2015

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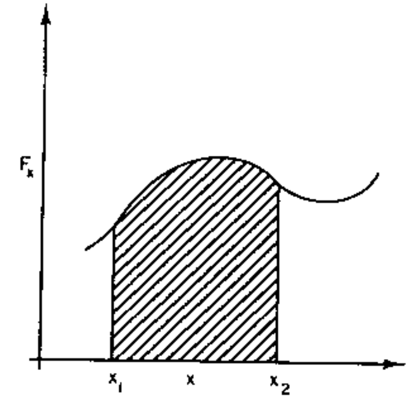
Work

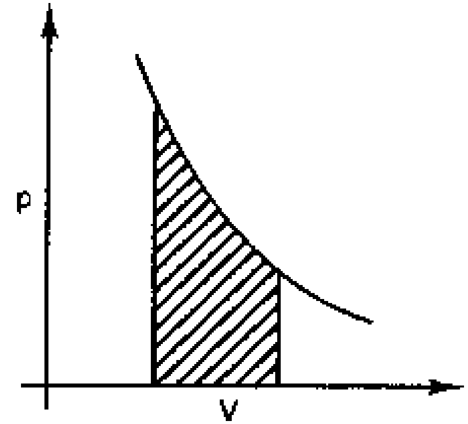
- Work done by a force F_x as it moves a solid from x_1 to x_2
  - Area under curve
  - Equal to increase in K.E.

$$W = \int_{x1}^{x2} F_x(x)dx$$

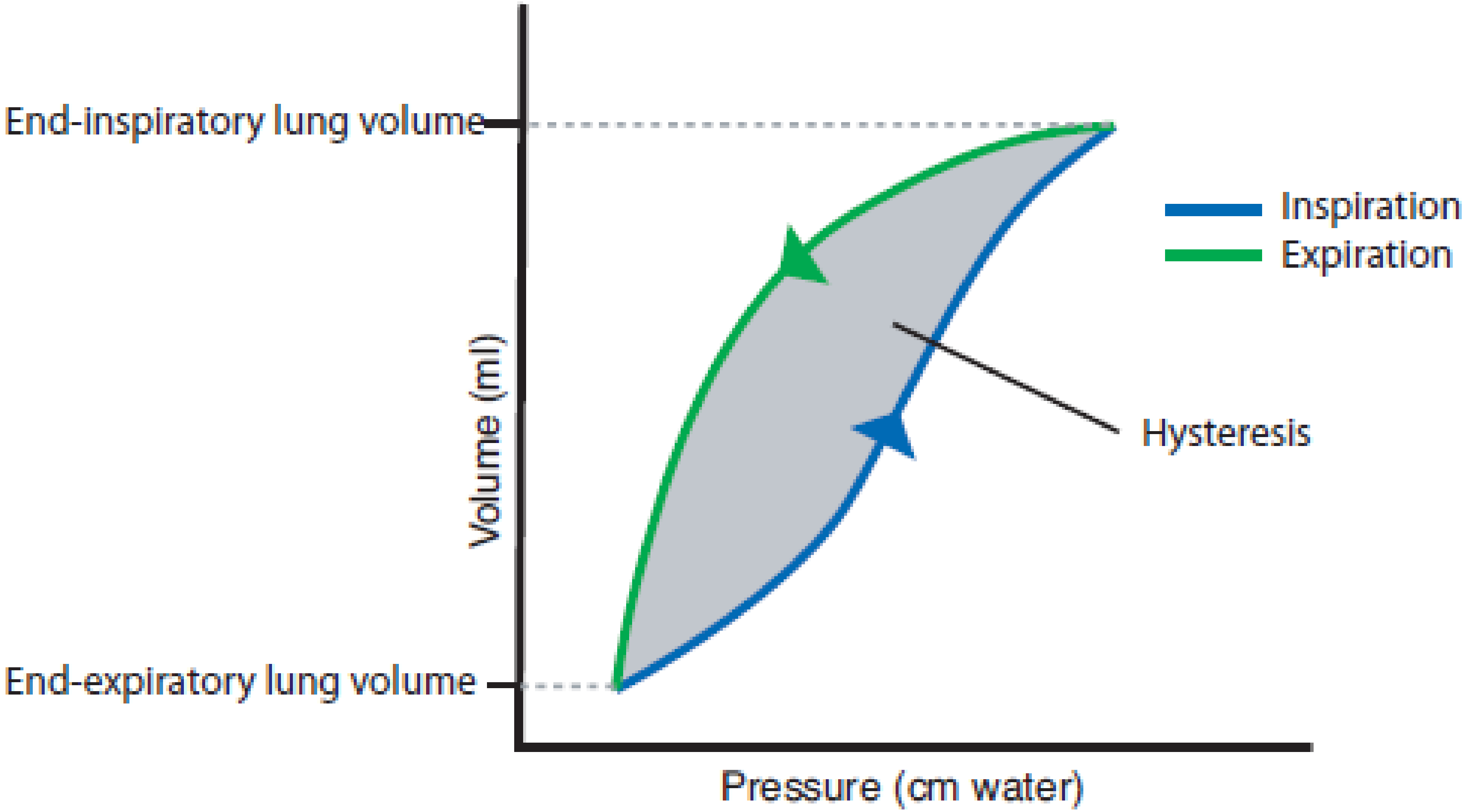


- Work done by a gas

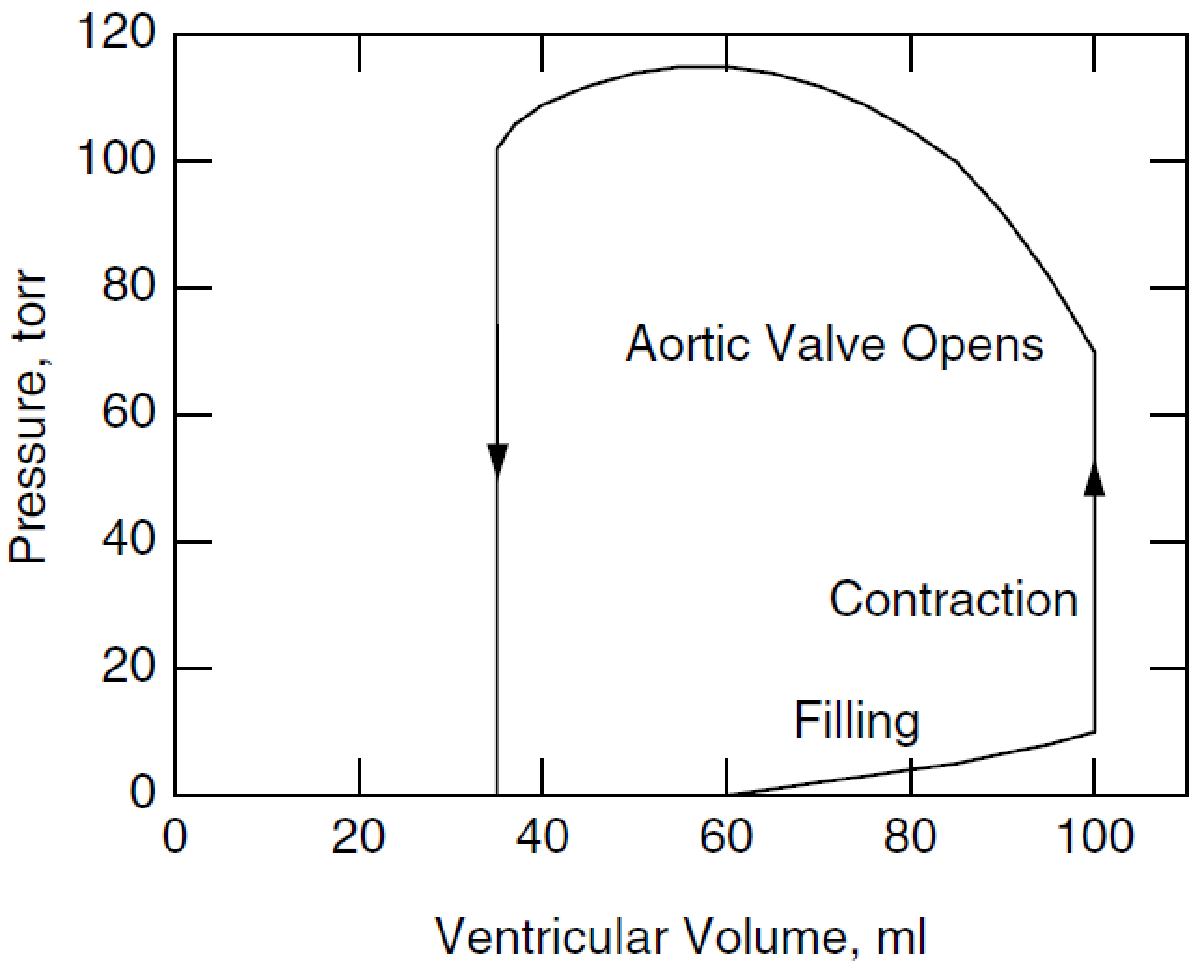
$$W_{by\,gas} = \int_{V1}^{V2} p \cdot dV$$



Example: Respiration Work



Example: Heart



Boltzmann Factor

- Relative probability of finding system A with energy U_r or U_s at absolute temperature T is given by

$$\boxed{R = e^{-(U_s - U_r)/k_B T}}$$

  - k_B is the Boltzmann constant

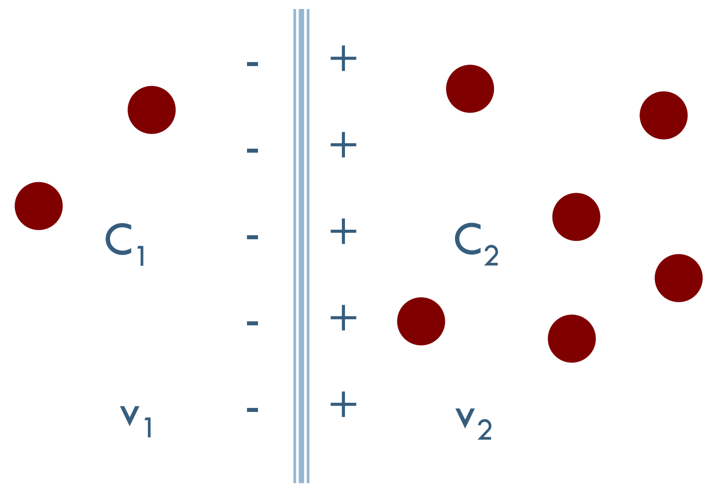
- Example: Atmospheric Pressure Variation with Altitude
  - Potential energy (gravitational) $=m\times g\times y$

$$\boxed{\frac{P(y)}{P(0)} = \frac{C(y)}{C(0)} = e^{-mgy/k_B T}}$$

Nernst Equation

- Concentration of ions on the two sides of a semi-permeable membrane and its relation to the voltage across the membrane

$$\frac{P(2)}{P(1)} = \frac{C_2}{C_1}$$



- $U = E_{kinetic} + E_{potential}$ ($E_{kinetic}$ *is the same*)
  - Potential energy is $E_p = zev$

$$\frac{C_2}{C_1} = e^{-ze(v_2 - v_1)/k_B T}$$

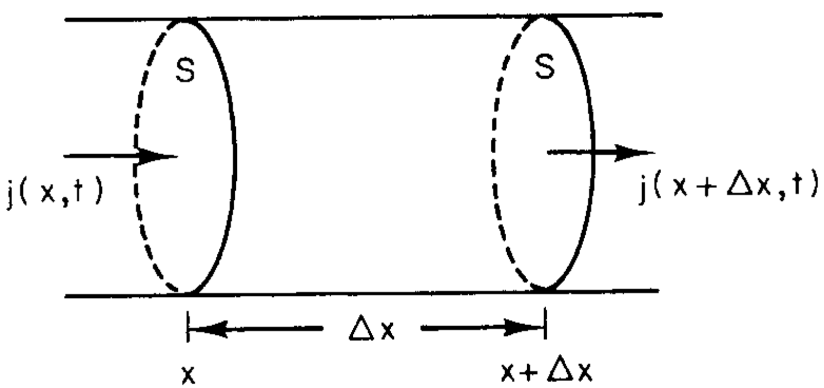
- Substitute $R = N_A K_B$ and $F = N_a e$ and rearranging, we get:

$$v_2 - v_1 = \frac{RT}{zF} \ln\left(\frac{C_1}{C_2}\right)$$

Continuity Equation

- We deal with substances that do not "appear" or "disappear"
  - "Conserved"
- Conservation of mass leads to derivation of continuity equation
  - Change in mass within a unit volume is related to difference between input and output to that unit volume
- 1D continuity equation:

$$\boxed{\frac{\partial C}{\partial t} = -\frac{\partial j}{\partial x}}$$



Brownian Motion

- Application of thermal equilibrium at temperature T
- Kinetic energy in 1D = $k_B T/2$
- Kinetic energy in 3D = $3k_B T/2$
- Random motion → mean velocity $\overline{v} = 0$
  - can only deal with mean-square velocity $\overline{v^2}$

$$\boxed{\frac{1}{2}m\overline{v^2} = \frac{3k_B T}{2} \Rightarrow v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}}}$$

Brownian Motion

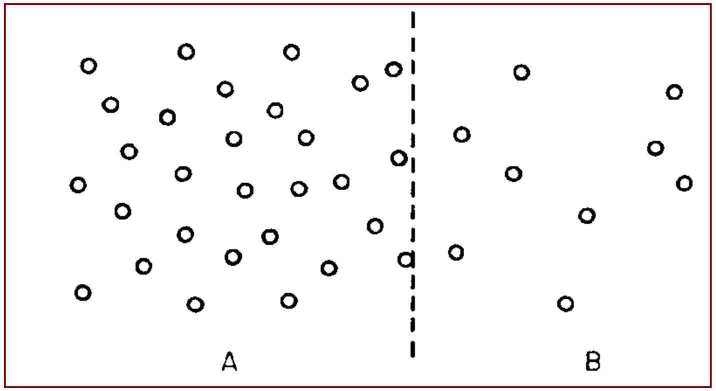
Particle	Molecular weight	Mass (kg)	v_{rms} (m s^{-1})
H_2	2	3.4×10^{-27}	1940
H_2O	18	3×10^{-26}	652
O_2	32	5.4×10^{-26}	487
Glucose	180	3×10^{-25}	200
Hemoglobin	65 000	1×10^{-22}	11
Bacteriophage	6.2×10^{6}	1×10^{-20}	1.1
Tobacco mosaic virus	40×10^{6}	6.7×10^{-20}	0.4
E. coli		2×10^{-15}	0.0025

Diffusion: Fick's First Law

- Diffusion: random movement of particles from a region of higher concentration to a region of lower concentration
  - Diffusing particles move independently
- If solute concentration is uniform, no net flow
- If solute concentration is different, net flow occurs

$$\boxed{j_x = -D\frac{\partial C}{\partial x}}$$

D: Diffusion constant (m^2s^{-1})



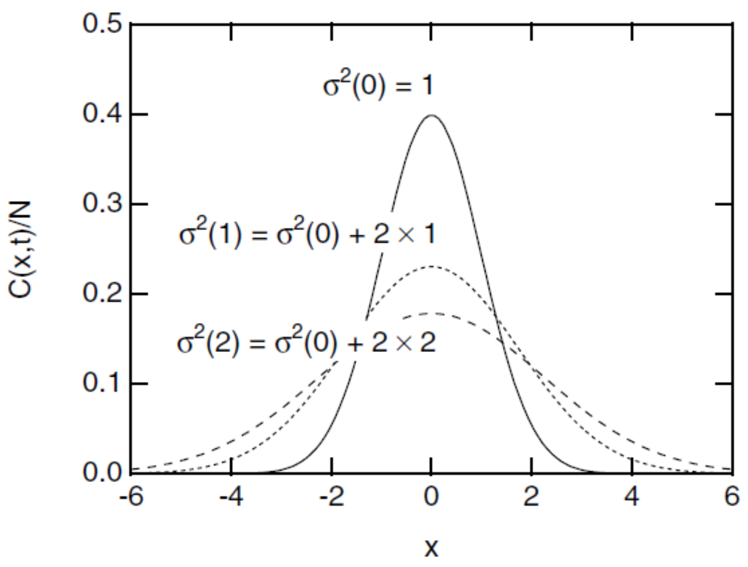
Diffusion: Fick's Second Law

- Derived by combining Fick's first law and continuity equation

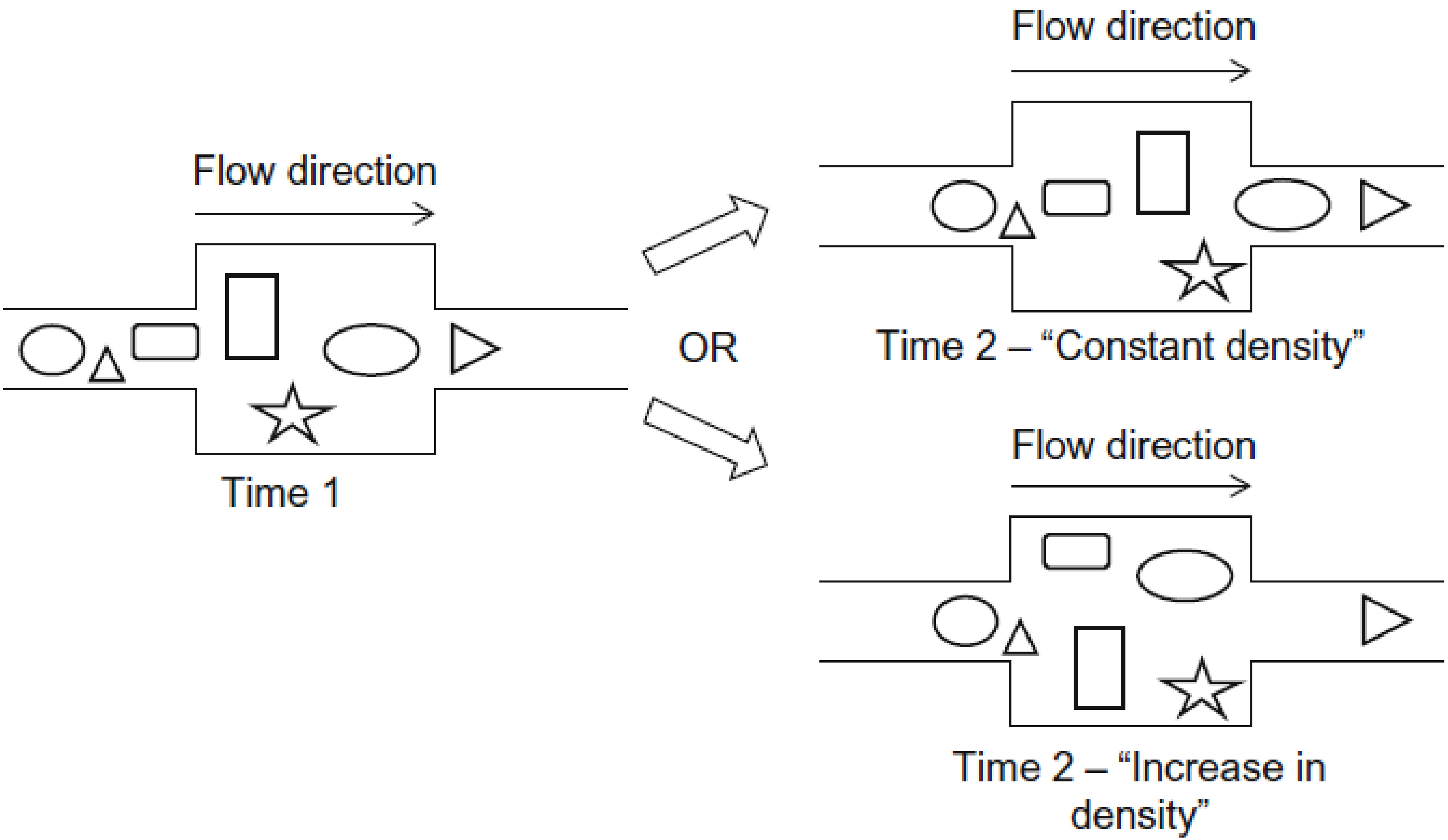
$$\boxed{\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}}$$

$$\boxed{C(x,t) = \frac{N}{\sqrt{2\pi\sigma(t)}}e^{-x^2/2\sigma^2(t)}}$$

$$\boxed{\sigma^2(t) = 2Dt}$$

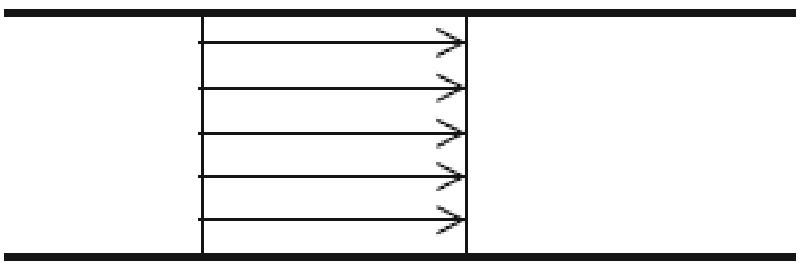


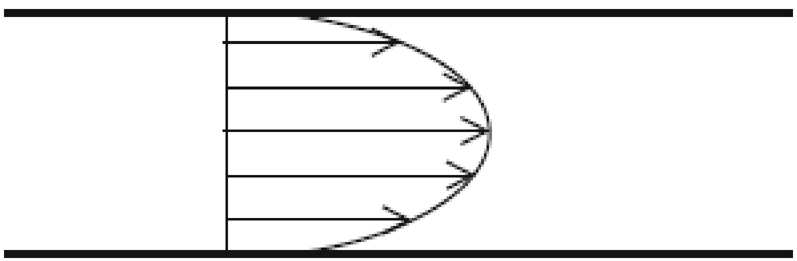
Compressible and Incompressible Fluids

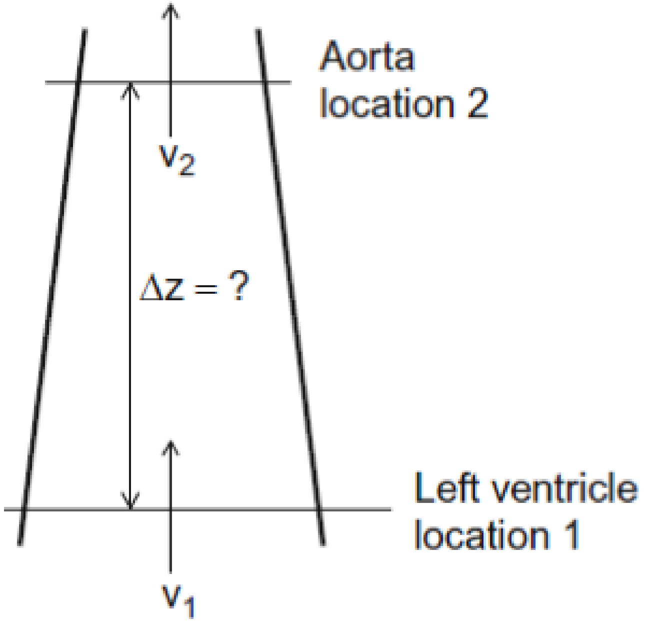


Mechanics of Incompressible Fluids

- Viscosity is equivalent to "friction" in solid mechanics
  - Fluid flow without (flat profile) and with viscosity (parabolic profile)







- Continuity equation

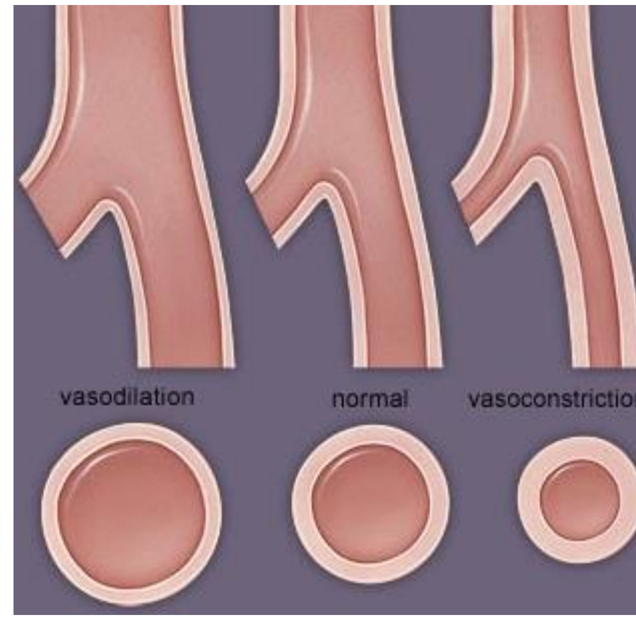
$$\boxed{A_1 v_1 = A_2 v_2}$$

  - The smaller the cross sectional area, the higher the flow velocity

Pressure, Flow and Vascular Resistance

- Analogous to Ohm's law in electricity

$$R = \frac{8\eta\Delta x}{\pi R_p^4} \qquad R = \frac{\Delta p}{i}$$

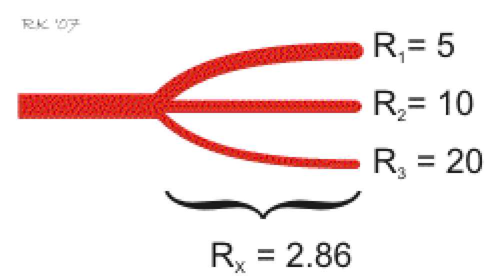


- Series Vessels

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \cdots$$

- Parallel Vessels

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$



$$R_x = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

Reynolds Number Measure of Flow Turbulence

Location	Average pressure (torr)	Blood volume[a] (ml)	Diameter[b] (mm)	Length[b] (mm)	Wall thickness[b] (mm)	Avg. velocity[b] (m s^{-1})	Reynolds number at maximum flow [c]
			Systemic circulation				
Left atrium	5						
Left ventricle	100						
Aorta	100	156	20	500	2.00	4.80×10^{-1}	9 400
Arteries	95	608	4	500	1.00	4.50×10^{-1}	1 300
Arterioles	86	94	0.05	10	0.2	5.00×10^{-2}	
Capillaries	30	260	0.008	1	0.001	1.00×10^{-3}	
Venules	10	470	0.02	2	0.002	2.00×10^{-3}	
Veins	4	2682	5	25	0.5	1.00×10^{-2}	
Vena cava	3	125	30	500	1.5	3.80×10^{-1}	3 000
Right atrium	3						
			Pulmonary Circulation				
Right atrium	3						
Right ventricle	25						
Pulmonary artery	25	52					
Arteries	20	91					7 800
Arterioles	15	6					
Capillaries	10	104					
Veins	5	215					2 200
Left atrium	5						

$$\boxed{N_R = \frac{LV\rho}{\eta}}$$

Nature of Light

- Light travels in a vacuum with a velocity $c = 3 \times 10^8\ m/s$
- When light travels through matter, its speed is less than this and is given by

$$\boxed{c_n = \frac{c}{n},}$$

  - *n is index of refraction of substance*
  - depends on both the composition of substance and color of light.

Nature of Light

- Controversy over the nature of light existed for centuries
  - Isaac Newton: particle model
  - Thomas Young: Interference experiments
  - Late 19th century: waves
  - Early 20th century: dual nature
- Wave Period T, frequency ν, and wavelength λ, are related as:

$$\boxed{\nu = \frac{1}{T}, \qquad c_n = \lambda \nu.}$$

- Each particle of light or photon has energy E given by:

$$\boxed{E = h\nu = \frac{hc_n}{\lambda}.}$$

Nature of Light

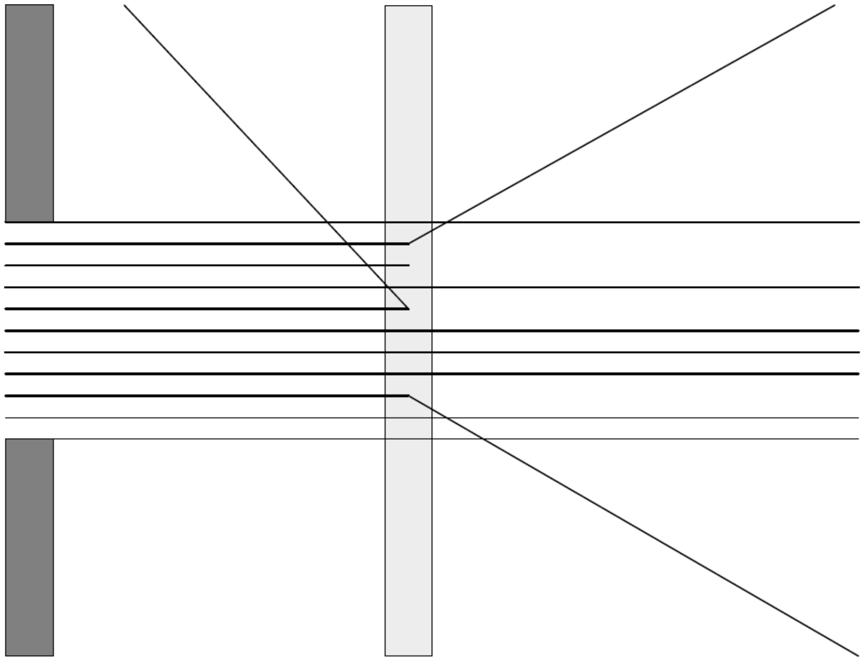
- As light moves from one medium into another where it travels with different speed, frequency remains the same
  - Wavelength changes as the speed changes

Name	Wavelength	Frequency (Hz)	Energy (eV)
Radio waves			
	1 m	300×10^{6}	1.24×10^{-6}
Microwaves			
	1 mm	300×10^{9}	1.24×10^{-3}
Extreme infrared			
	15 μm	20×10^{12}	0.083
Far infrared			
	6 μm	50×10^{12}	0.207
Middle infrared			
	3 μm	100×10^{12}	0.414
Near infrared			
	750 nm	400×10^{12}	1.65
Visible			
	400 nm	750×10^{12}	3.1
Ultraviolet			
	12 nm	24×10^{15}	100
X rays, γ rays			

Scattering and Absorption of Radiation

- Imagine that we have a distant source of photons that travel in straight lines, and that we collimate the beam
  - a nearly parallel beam of photons
  - Imagine also that we can see the tracks of the N photons in the beam

- Passing through
- Scattering
- Absorption



Scattering and Absorption of Radiation

- Assume N photons passing through a thin layer of material *dz*

$$dN_s = \mu_s N dz, \qquad dN_a = \mu_a N dz.$$

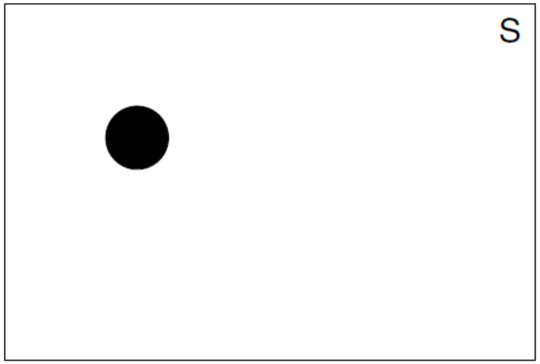
$$dN = -(dN_s + dN_a) = -N(\mu_s + \mu_a)dz$$

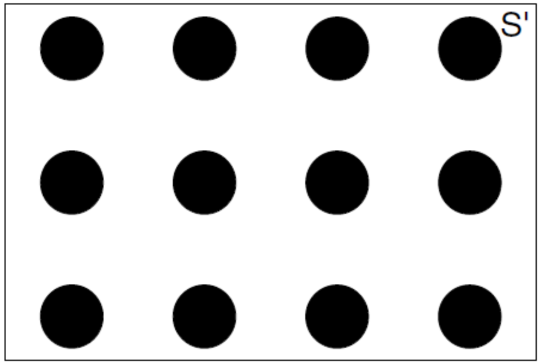
$$\boxed{N(z) = N_0 e^{-\mu z} = N_0 e^{-(\mu_s + \mu_a)z}.}$$ **Beer's Law**

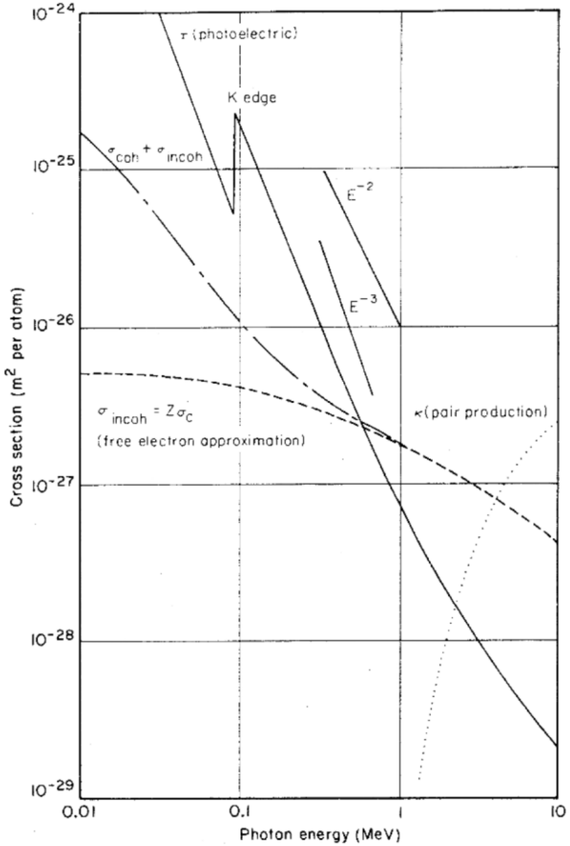
  - *μ is the total linear attenuation coefficient.*
  - *μ_s and μ_a are linear scattering and absorption coefficients*

Interaction Cross Section

- Interaction of photons with matter is statistical
- The *cross section σ* is an effective area proportional to the probability that an interaction takes place
- Used in describing X-ray interactions







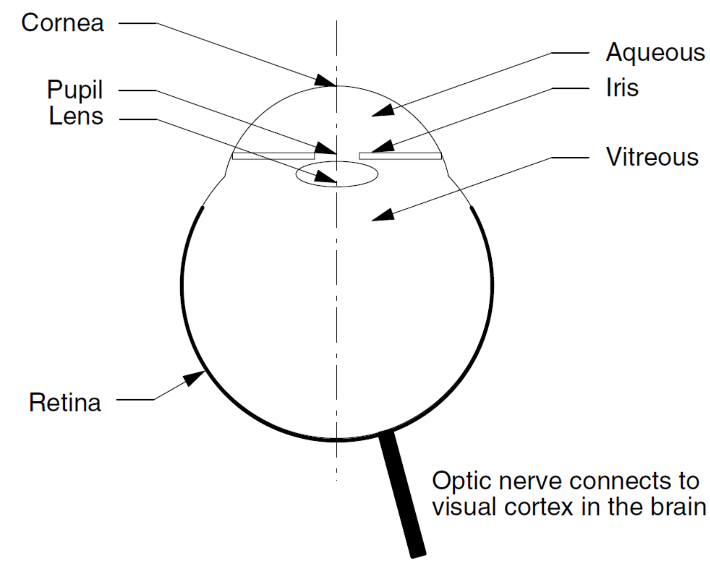
The Eye

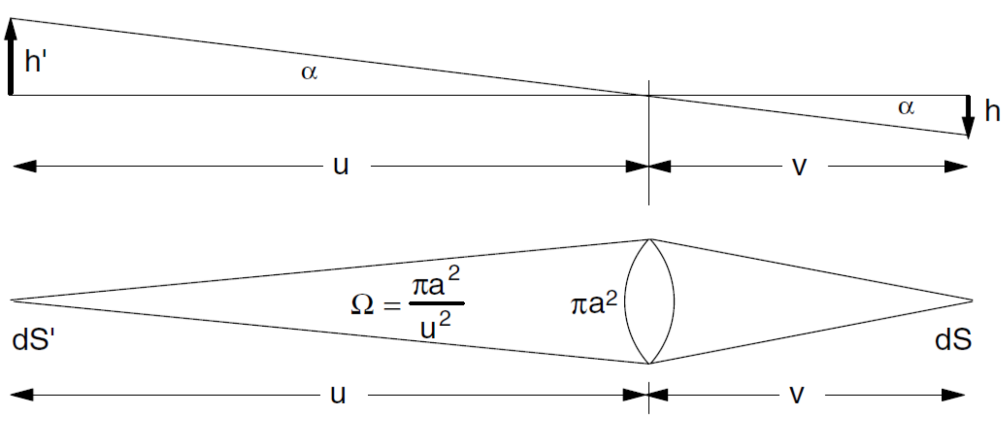
- Snell's law

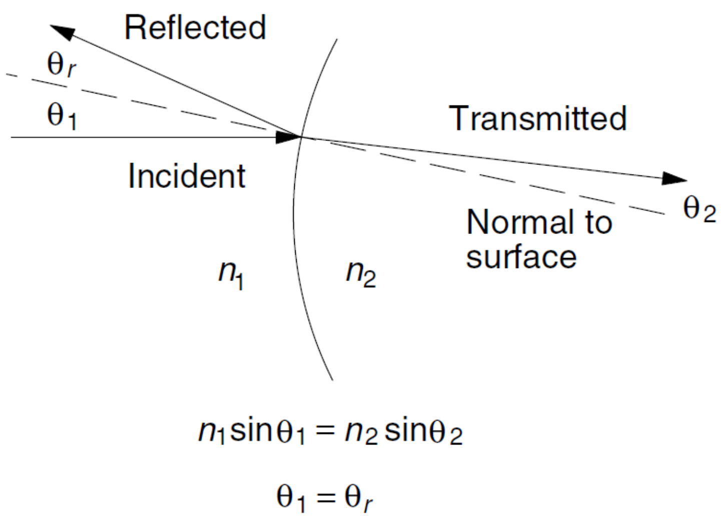
$$\boxed{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$

- Thin lens equation

$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$







The Eye

- Optometry: Vergence (diopters)

$$\boxed{\begin{aligned} U &= -\frac{1}{u} \quad \text{(diverging from the object)}, \\ V &= \frac{1}{v} \quad \text{(converging to the image)}, \\ F &= \frac{1}{f} \quad \text{(a converging lens)}. \end{aligned}}$$

$$\boxed{V = F + U.}$$

The Eye

- Accommodation
  - Decreases with age: bifocals
- Emmetropic (normal) eye: $V=F$ (when $U=0$)
- Nearsightedness or myopia: $F>V$
- Farsighted or hypermetropic: $F<V$
- Astigmatism
  - Eye is not symmetric about an axis through center of lens
- Chromatic aberration
  - Index of refraction of lens varies with wavelength.
- Spherical aberration
  - Refractive power changes with distance from the axis of the eye.

Suggested Readings and Assignments

- Suggested supplementary textbook for more information:

R. Hobbie and B. Roth, Intermediate Physics for Medicine and Biology, 2007.

- Homework posted on web site