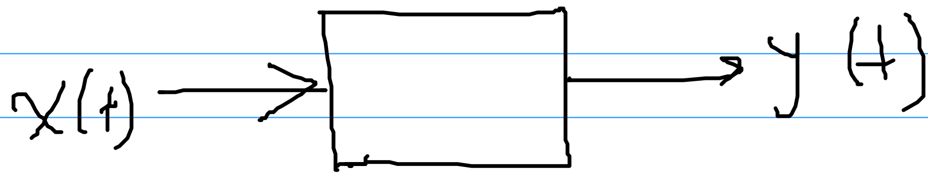
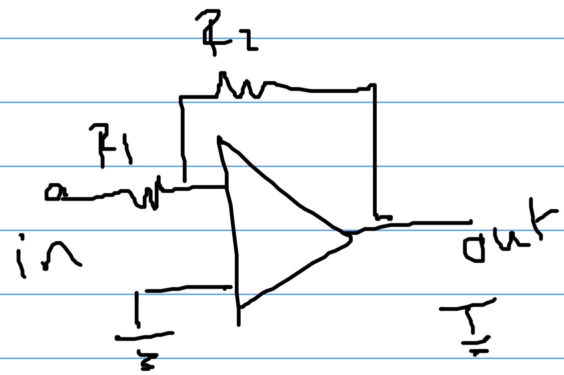
# Scaling



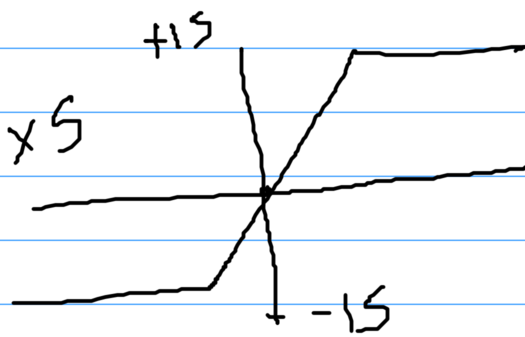

$a\ x(t)$ $\qquad$ $a\ y(t)$




$= \frac{-R_2}{R_1}\, in$

1V → 10V

5V → 15 $\neq$ 10×5




$y = \underline{ax + b}$

replace $x$ with $(\alpha x)$

and check if output $= \alpha y$

$a\alpha x + \overset{0}{\cancel{b}} \neq \alpha y$ ?

$=$ $\qquad$ $a\alpha x + \cancel{\alpha} b$

$$x \longrightarrow \boxed{\quad\quad} \longrightarrow y$$

$x_1$ $\quad\quad\quad$ $y_1$

$x_2$ $\quad\quad\quad$ $y_2$

$x_1 + x_2$ $\quad\quad\quad$ $\overset{?}{}\ y_1 + y_2$

$$y = \frac{dx}{dt} + x$$

$$x_1 \longrightarrow y_1 = \frac{dx_1}{dt} + x_1$$

$$x_2 \longrightarrow y_2 = \frac{dx_2}{dt} + x_2$$

$$(x_1 + x_2) \longrightarrow \text{output} = \frac{d(x_1+x_2)}{dt} + (x_1 + x_2)$$

$$= \underline{\underline{\frac{dx_1}{dt}}} + \underline{\underline{\frac{dx_2}{dt}}} + \underline{x_1} + x_2 =$$

$$= y_1 + y_2 \Rightarrow \checkmark$$

$$y(t) = x(2t)$$

Scaling

$$\text{output} = \alpha x(2t) = \alpha y \checkmark\checkmark$$

additivity

$$y_1 = x_1(2t)$$

$$y_2 = x_2(2t)$$

$$y(x_1 + x_2) = x_1(2t) + x_2(2t)$$

$$= y_1 + y_2 \checkmark\checkmark$$

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$$y(t) = \int_{-\infty}^{\infty} \underline{[u(\tau) - u(\tau - 1)]}\, x(t - \tau)\, d\tau$$

Scaling

additivity

$$y_1 = \int \quad x_1$$

$$y_2 = \int \quad x_2$$

$$\int \quad (x_1 + x_2)$$

$$y(t) = \int_{-\infty}^{t} a\, x_{\substack{1\\2}}(\tau)\, d\tau$$

$x_1$

$$\int_{-\infty}^{t} \left[x_1(\tau) + x_2(\tau)\right] d\tau$$

$$y_1 + y_2$$

$$y = \sin(x)$$

$$\left(\frac{dx}{dt}\right)^2 + 5$$

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$y(t) = a\,x(t) + b$ $\qquad t \to t - \tau$

$x(t) \to \boxed{x(t-\tau)}$

$a\,x(t-\tau) + b = \boxed{y(t-\tau)}$

$$y(t) = t\,\frac{dx}{dt} \qquad t = 0 \qquad t = 1$$

$$x(t-\tau) \longrightarrow \text{output} = t\,\frac{dx(t-\tau)}{dt} \quad ✗$$

$$y(t-\tau) = (t-\tau)\,\frac{dx(t-\tau)}{dt}$$

$$\frac{d(\tau - t)}{dt} = -dt \qquad d(t - \cancel{\tau}) = dt$$

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$$y(t) = x(2t)$$

$$x(t-\tau) \Rightarrow x(2(t-\tau))$$

$$y(t-\tau) \Rightarrow x(2(t-\tau)) \quad ✓✓$$

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$$y(t) = \cos(\Omega_0 t)\, x(t)$$

$$x(t-\tau) \Rightarrow \cos(\Omega_0 t)\, x(t-\tau) \quad ✗$$

$$y(t-\tau) = \cos(\Omega_0 (t-\tau))\, x(t-\tau)$$

$$y(t) = \underline{x(t/2)}$$

$x=0 \quad y=0$ ✓

$y(t)$ depend value of $x$ $t < t$

$$\underline{\underline{y(1)}} = \underline{\underline{x(2\times 1)}} \quad \times$$

$$y(-1) = x\left(-\frac{1}{2}\right) \quad \times$$

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$$y = \int_{-\infty}^{t} \underbrace{x(\tau)}\, d\tau < \infty$$

$x(\tau) = 1 \qquad x(\,) < \infty$