$$x(t) \xrightarrow{\mathcal{F}} X(\Omega)$$

Duality

$$X(t) \longrightarrow 2\pi\, x(-\Omega)$$

$$\text{Periodic} \xrightarrow{\mathcal{F}_c} \text{sampled}$$

$$\text{Sampled} \qquad \text{periodic}$$

Time shift

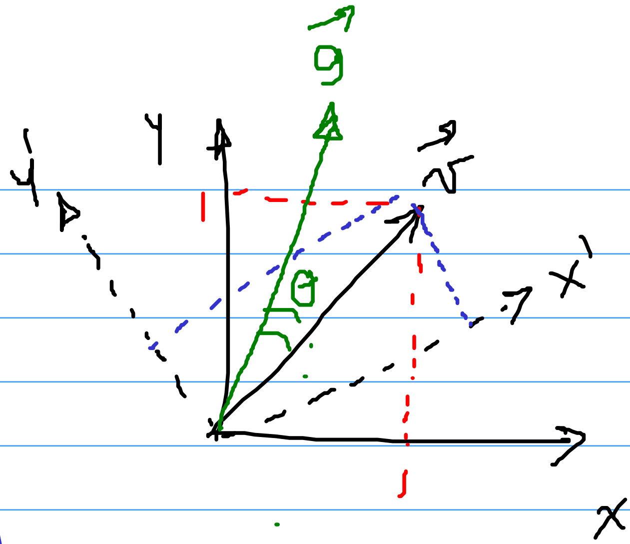
$$x(t-\tau) \qquad e^{-j\Omega\tau} X(\Omega)$$

Mod.

$$e^{j\Omega_0 t} x(t) \qquad X(\Omega - \Omega_0)$$

$$x(at) \qquad \frac{1}{|a|} X\left(\frac{\Omega}{a}\right)$$

$$x * h \qquad X(\Omega) \cdot H(\Omega)$$



(x, y)

(x', y')

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \boxed{} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Orthogonal Transformation

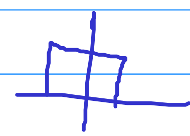
1. Length
2. Angle

Ex: Show whether the following signal is finite energy or finite power

$$x(t) = \frac{\sin(t)}{\pi t}$$

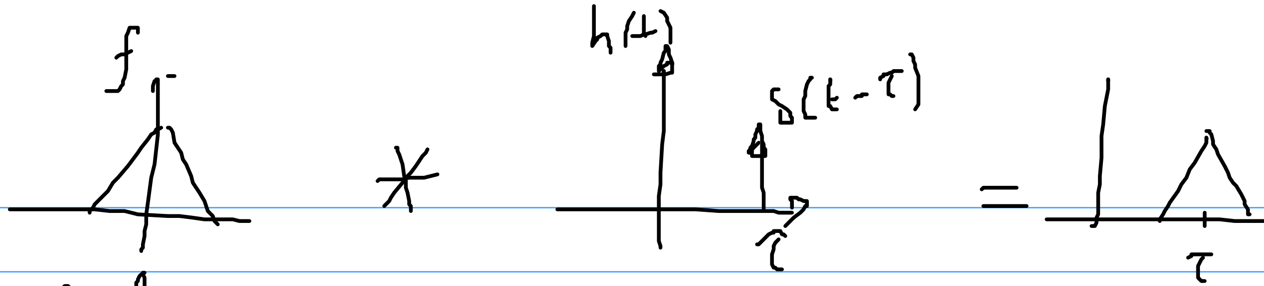
Solution: $$E = \int_{-\infty}^{\infty} \left(\frac{\sin(t)}{\pi t} \right)^2 dt$$

From Parseval's theorem:



$$E = \int_{-\infty}^{\infty} \left[u(\Omega + 1) - u(\Omega - 1) \right]^2 d\Omega$$

$= \text{Finite} \implies$ finite energy!



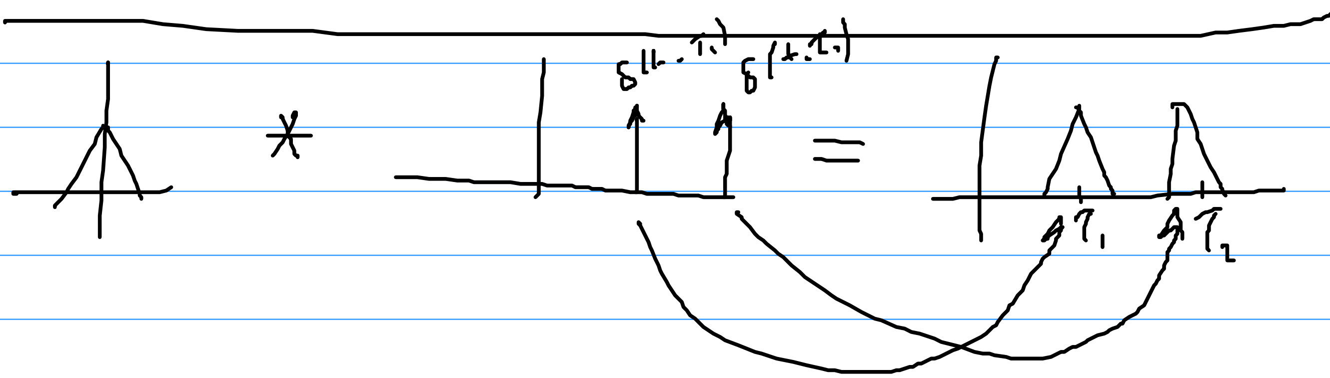
$$\int_{-\infty}^{\infty} f(\alpha)\, h(t-\alpha)\, d\alpha$$

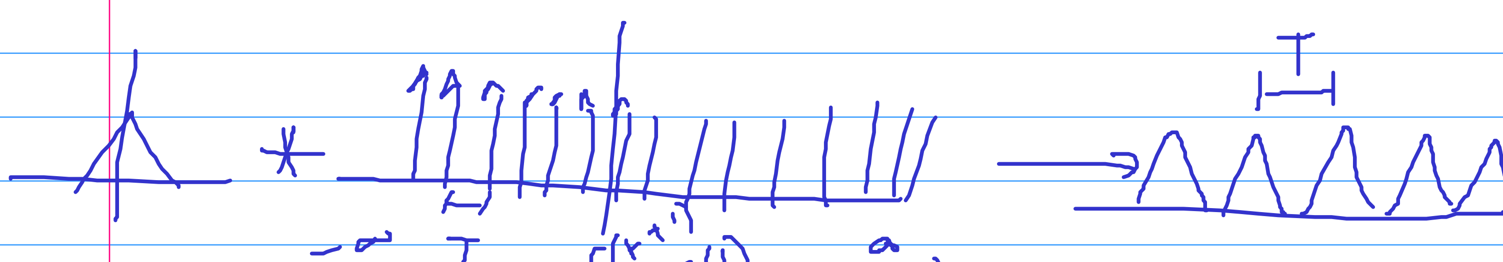
$$= \int_{-\infty}^{\infty} f(\alpha)\, \underbrace{\delta(t-\alpha-\tau)}_{t-\alpha-\tau=0}\, d\alpha$$

$$\alpha = t - \tau$$

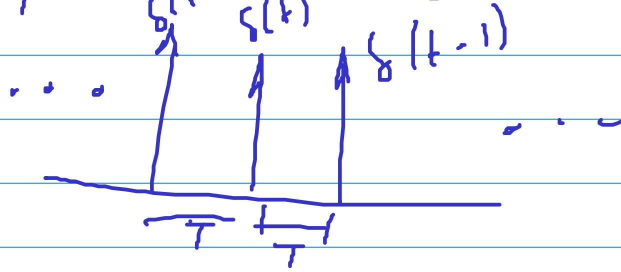
$$= f(t-\tau)$$

1

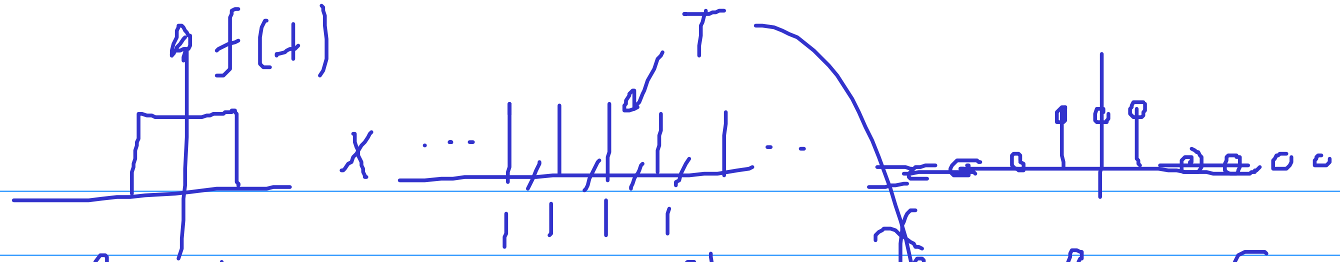




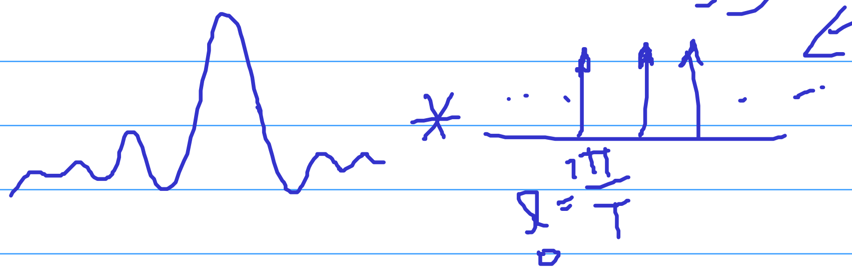
Comb



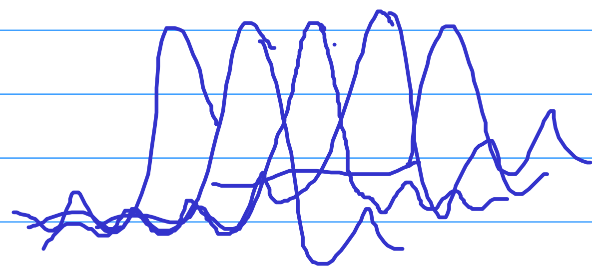
Sampling function

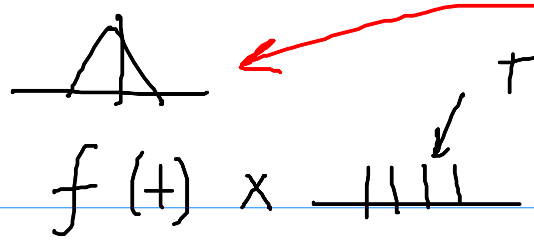


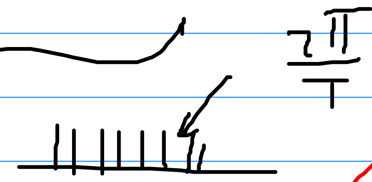
function $\times$ Sampling $\Longleftrightarrow$ $f * Sa$

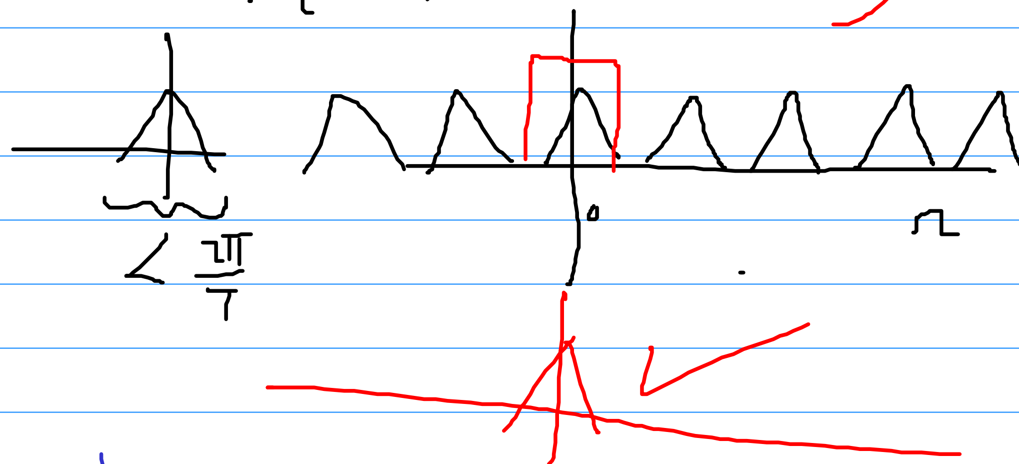


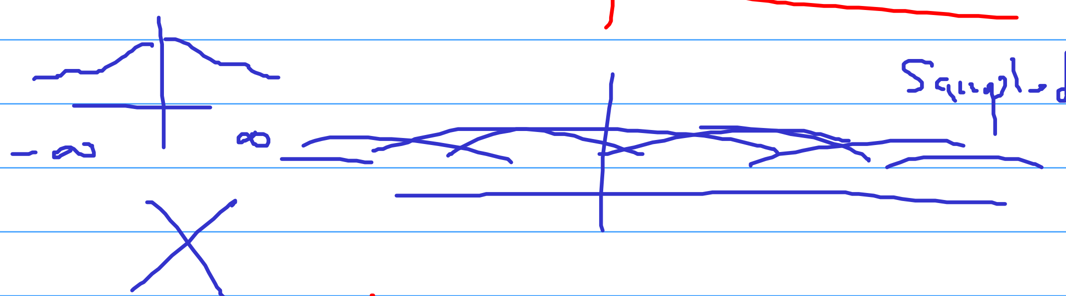
$\Omega_0 = \frac{2\pi}{T}$



Sampling $\rightarrow$ $\underbrace{f(t) \times}$ 

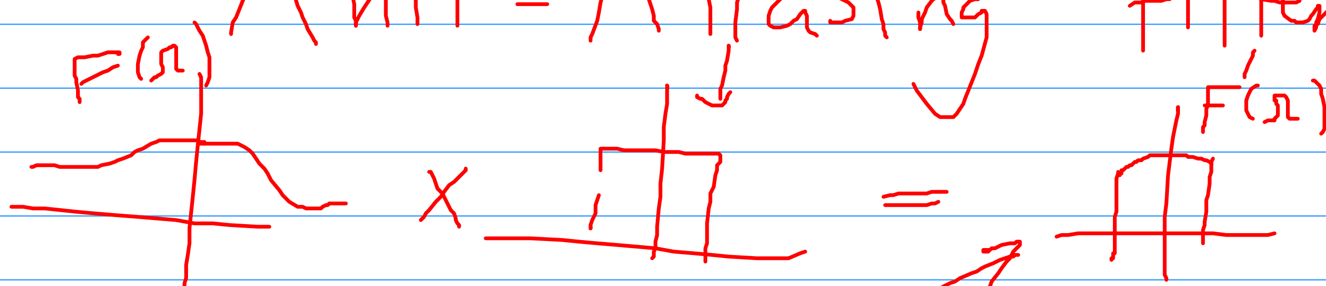
$F(\Omega) *$ 

$F(\Omega)$ 

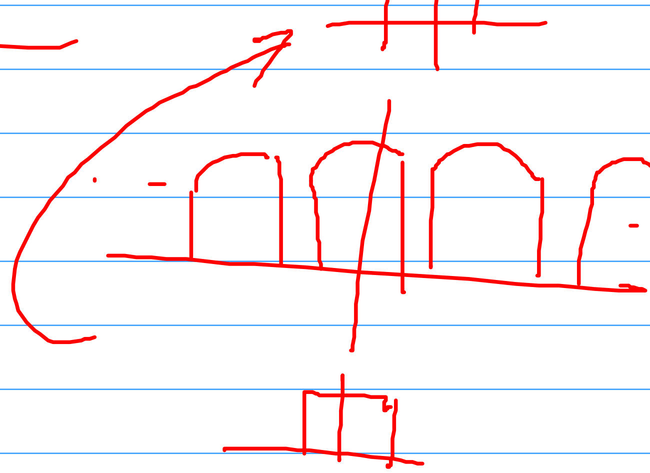
$F(\Omega) \Rightarrow$ 

Solution in practice :

Anti-Aliasing Filter



after sampling



Aliased Signal Cannot be Recovered !