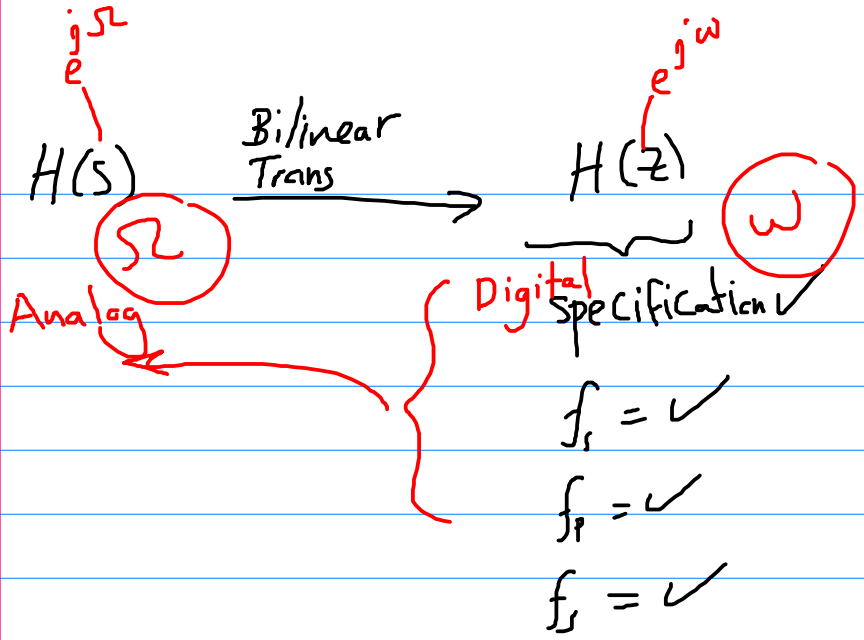


Frequency Warping

$$\underbrace{\Omega}_{\text{analog}} = \underbrace{\frac{2}{T_s}}\ K \tan\left(\underbrace{\frac{w}{2}}_{\text{digital}}\right)$$

Ex: $H(s)$ LPF, $f_p = 1\,KHz$

Bilinear $f_s =$ 10 KHz $\longrightarrow w_p = ?$

$f_s = \frac{1}{T_s}$

$$2\pi(1000) = 2(10000) \times \tan\left(\frac{w_p}{2}\right)$$

$w_p = ?$

Frequency Scale

$f_p = 1000$ $\qquad$ $f_s = 10\,000$

$\downarrow$

$w_p = ?$

	f_p	
0	1000	$0.5 \times 10\,000$
0	$w_p = ?$	π

$$w_p = \frac{\pi}{5\,000} \times 1000 = 0.2\pi$$

Ex Digital filter $\omega_p = 0.1\pi$, $f_s = 100$ Hz

$\longrightarrow f_p = ?$

0	f_p	0.5×100
0	0.1π	π

$$f_p = \frac{0.5 \times 100}{\pi} \times 0.1\pi = 5 \text{ Hz}$$

Design Using Bilinear Transformation

(1) Specs Digital filter ✓

$\omega_p = $ ✓ $\omega_{st} = $ ✓ $f_s = $ ✓

(2) Use Freq Warping to get specs for analog filter

$$\Omega = K \tan\left(\frac{\omega}{2}\right)$$

$f_p = $ ✓ $f_{st} = $ ✓

(3) Design (look up) analog filter

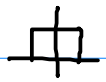
$H(s) = $ ✓

(4) Apply Bilinear Trans.

$$s = K \frac{1 - z^{-1}}{1 + z^{-1}}, \quad K = \frac{2}{T_s} = 2 f_s$$

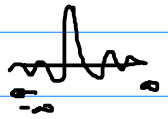
$H(s) \longrightarrow H(z)$

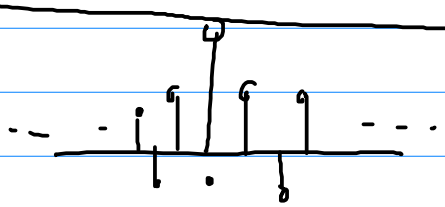
Frequency Transformation

① $H(\Omega)$ = ✓ 

② $H(\Omega) \xleftrightarrow{\mathcal{F}^{-1}} h(t)$

③ Sample $h(t)$ with f_s

$h(n) \rightarrow$ IIR 

Window 

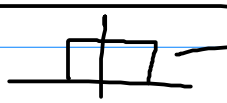
X X ⌊——⌋ X X

M points

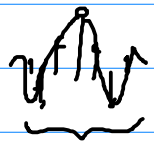
Fixed M
User

As many points as needed to meet specs

Practice 5

H_d = ✓  $\xrightarrow{\mathcal{F}^{-1}}$ ① $\text{Sinc}(\omega_0 t)$

Sampling ② $t = \frac{1}{f_s} n = nT_s$

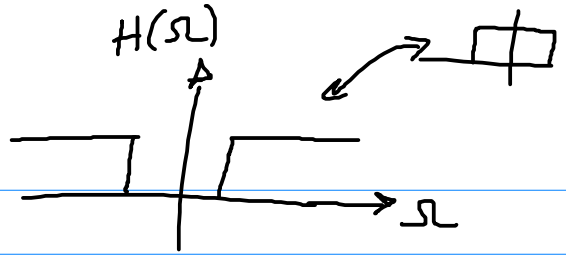
Response $h(n)$ $\text{Sinc}(\omega_0 T_s n)$ 

FIR M=7

filter = ↓ ③ with $n=-3:3$

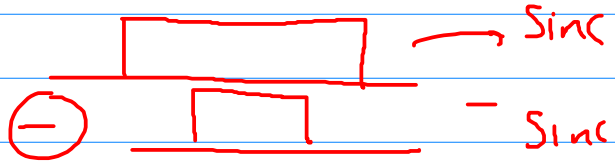
M=23 $-11:11$ M=24

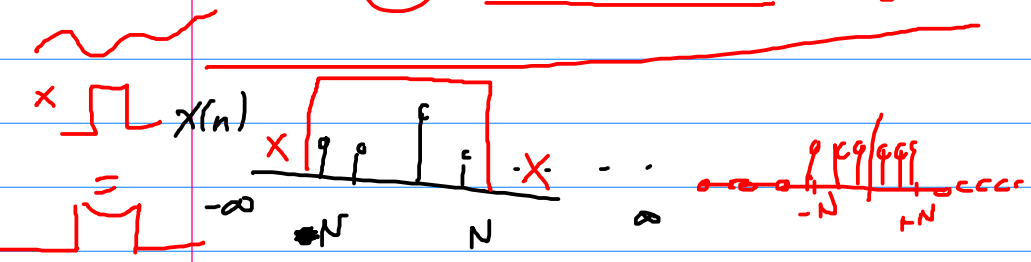
Ex:



$= 1 -$

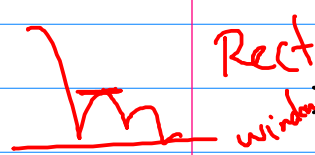
Ex



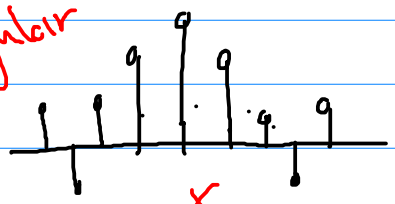


$x(n) \times w(n)$

$\hookrightarrow X(e^{j\omega}) * W(e^{j\omega})$

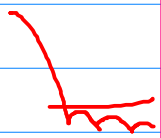


Rectangular window

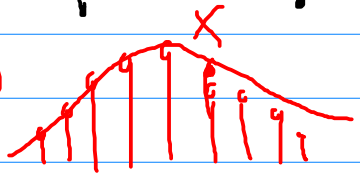


$M = 9$

ok filter



Window



= Better Filter

Realization

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 - z^{-1} - z^{-2}} = \frac{Y(z)}{X(z)}$$

$$y(n) = x(n) + x(n-1) + x(n-2) + y(n-1) + y(n-2)$$