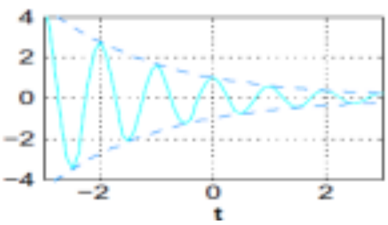
Solve As Much As You Can – Maximum Grade: 100 Points

Q1. For each signal, determine if it is periodic, and if it is, find the fundamental period [5 Points Each]:

(a) x(t) is as shown in the figure



Since $x(t+T) \neq x(t)$ for any T,

Then, Not Periodic

(b) x(t)= 2 cos(πt+π/3) + 3 sin(3πt/2-1)

periodic $T_1 = \text{period} = \frac{2\pi}{\pi} = 2$ periodic $\text{period} = \frac{2\pi}{3\pi/2} = \frac{4}{3} = T_2$

$T_1/T_2 = \frac{2}{4/3} = \frac{6}{4} = \frac{3}{2}$ Rational ⟹ x(t) Periodic

Period of x = Least Common Multiple of 2, 4/3 = 4

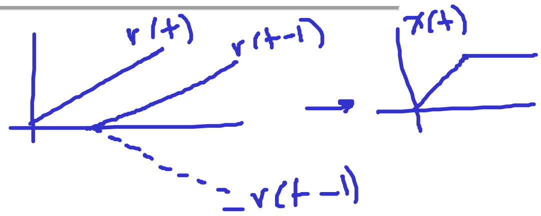
Q2. Categorize each of the following signals as a finite energy signal or a finite power signal, [5 Points Each]:

(a) x(t)= $100\sin(100\pi t))/t$ $\xrightarrow{F} A[u(\Omega+100\pi) - u(\Omega-100\pi)]$

$E = \int_{-\infty}^{\infty} |x(t)| dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\Omega)|^2 d\Omega$ (Parseval) $< \infty$

⟹ Finite energy

(b) x(t)= r(t) – r(t-1) (recall that r(t) is the ramp signal)



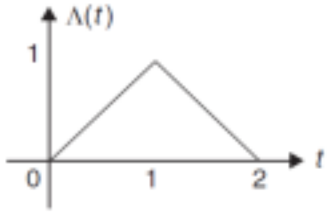
Since $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$

Then infinite energy ⟹ finite power

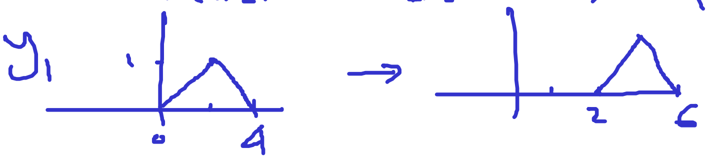
Q3. For the shown triangular pulse $\mathbf{\Lambda}(t)$, sketch each of the following signals [5 Points Each]:

(a) $y_1(t) = \mathbf{\Lambda}(t/2-1)$ $= \Lambda((t-2)/2)$

(b) $y_2(t) = 2\,\mathbf{\Lambda}(-2t+1)$



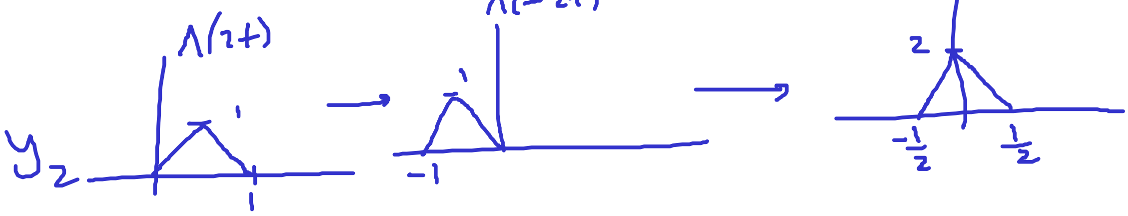
$\Lambda(t/2)$ $\quad \Lambda((t-2)/2) = y_1$



$2\,\Lambda(-2(t-1/2))$

$\Lambda(-2t)$

$\Lambda(2t)$



Q5. Determine whether the following signals are even or odd [5 Points Each]:

(a) $x(t) = \sin(2t) + \cos(3t^2)$

$x(-t) = -\sin(2t) + \cos(3t^2) \neq x(t)$

$\longrightarrow$ not even or odd

(b) $x(t) = t \sin(t)$

$x(-t) = -t \times (-\sin(t)) = t \sin(t) = x(t)$

$\longrightarrow$ even

Q4. For each system, determine whether it is (i) linear, (ii) time invariant and (iii) causal [6 Points Each]:

(a) $\underbrace{y(t) = \int_{-\infty}^{\infty} e^{-2\tau-1}u(\tau)\cdot x(t-\tau)d\tau}_{\text{looks like Convolution}} = \left[\underbrace{e^{-2t-1}u(t)}_{h(t)}\right] * x(t)$

$\Rightarrow$ Then, system is Linear & time invariant

Causality: looking at impulse response $h(t)$, we find that $h(t)=0$ for $t<0$

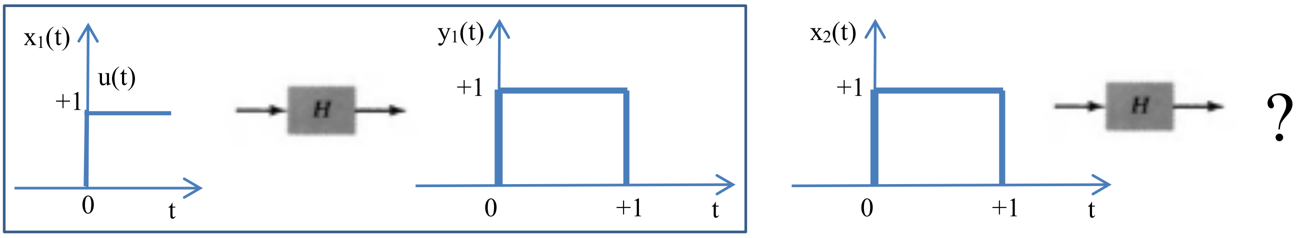
Then, system is causal

(b) $y(t) = \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x(t)$

Since system is represented by a linear diff. equation with Const. Coeff. & assuming zero initial conditions, $\Longrightarrow$ Linear & Time Inv.

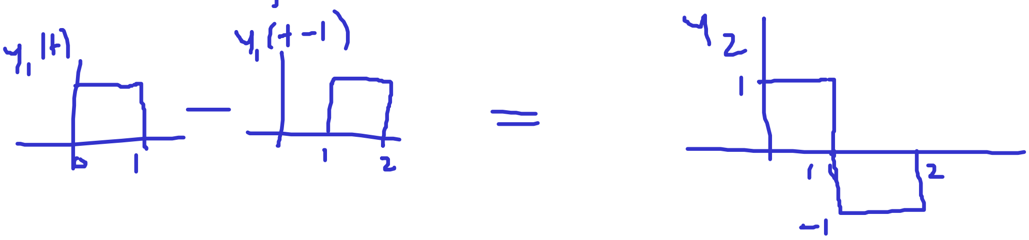
Since $y(t)$ depends only on x and its derivatives at t, Then system is Causal

6. For a linear time invariant (LTI) system, if the output of the system $y_1(t)$ is known for a particular input $x_1(t)$ (unit step function) as shown, compute the output of the same system for an input $x_2(t)$ shown. [10 Points]



Since $x_2(t) = u(t) - u(t-1) = x_1(t) - x_1(t-1)$

Then output $y_2(t) = y_1(t) - y_1(t-1)$ (since system is LTI)



7. Is it possible to compute the transfer function and impulse response for the systems defined by the following differential equation? Derive the formula for them if possible [10 Points].

$$y''(t) + 3y'(t) + 2y(t) = 2x'(t) + x(t) \qquad , with\ \ y(0) = 0, y'(0) = 0, x(0) = 0$$

zero initial cond. ✓

$$Y(s)\,[s^2 + 3s + 2] = X(s)\,[2s + 1]$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{2s+1}{s^2+3s+2} = \frac{A_1}{(s+2)} + \frac{A_2}{(s+1)}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = A_1 e^{-2t} u(t) + A_2 e^{-t} u(t)$$

8. Determine the unilateral Laplace transform of the following signals and systems [5 Points Each]:

(a) $x(t)= e^{-2t+1}\sin(100\,t)\,u(t)$

From Table: $X(s) = e \times \frac{100}{(s+2)^2+(100)^2}$

(b) $x(t)= e^{-t}\,[u(t) - u(t-1)]$ $= \underbrace{e^{-t}u(t)}_{\text{table}} + \underbrace{e^{-t}u(t-1)}$

$= e^{-1}e^{-(t-1)}u(t-1)$ table + time delay

$X(s) = \frac{1}{s+1} + \frac{1}{s+1}e^{-s}$

9. Determine the causal inverse of the following Laplace transformations [5 Points Each]:

(a) $X(s) = \frac{s^2+1}{s^2+5s+6}$, $ROC: \mathcal{R}e[s] > -3$

$= \frac{(s^2+5s+6)-5s-6+1}{s^2+5s+6} = 1 + \frac{-5s-5}{\underbrace{s^2+5s+6}_{(s+3)(s+2)}} = 1 + \frac{A_1}{s+3} + \frac{A_2}{s+2}$

$x(t) = \delta(t) + A_1 e^{-3t}u(t) + A_2 e^{-2t}u(t)$

(b) $X(s) = \frac{1}{s^2+9}$, $ROC: whole\ s\ plane$

From table: $x(t) = \frac{1}{3}\sin(3t)\,u(t)$

10. Compute the Fourier transformation of the following signals [5 points each]:

(a) $f(t) = e^{-2t}\cos(1000\,t)\,u(t)$ $\longrightarrow$ $= e^{-2t}u(t)$ modulated by $\cos(1000t)$

$$e^{-2t}u(t) \xrightarrow{\mathcal{F}} \frac{1}{j\Omega + 2}$$

$$F(\Omega) = \frac{1}{2}\left[\frac{1}{j(\Omega+1000)+2} + \frac{1}{j(\Omega-1000)+2}\right]$$

(b) $f(t) = \frac{1}{1+t^2}$

using duality property,

$$F(\Omega) = \frac{1}{2}\,e^{-|\Omega|}$$

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11. Compute the inverse Fourier transformation of the following spectra [5 points each]:

(a) $F(j\Omega) = \frac{\sin(\Omega-1)}{(\Omega-1)}$ $\longrightarrow$ sinc shifted by 1

$$\longrightarrow \quad f(t) = \left[u(t+1) - u(t-1)\right] e^{-j\Omega}$$

(b) $F(j\Omega) = u(\Omega-1)\,e^{-\Omega+3}$ $= e^{-(\Omega-1)}\,u(\Omega-1) \times e^{2}$

using duality property:

$$f(t) = e^{2}\,\frac{1}{-j\Omega+1}$$