EE470 – BIOMEDICAL SIGNALS AND SYSTEMS EXAM #2

Solve As Much As You Can – Maximum Grade: 100 Points

Q1. For each signal, determine if it is periodic, and if it is, find the fundamental period [5 Points Each]:

(a) $x[n]= \exp(-2\pi n)$

This is a decaying exponential

$\Longrightarrow$ not periodic

(b) $x[n]= 2\cos(-\pi n+\pi/3) + 3\sin(3n/2-1)$

$\frac{2\pi m}{N} = \pi$

$m=1, N=2$

periodic with period $= N = 2$

$\frac{2\pi m}{N} = \frac{3}{2}$

$\Downarrow$

not periodic

Sum is not periodic

Q2. Determine whether the following discrete signals are even or odd [5 Points Each]:

(a) $x[n]= n\sin(2n) + \cos(3n^2)$

$x[-n] = -n(-\sin(2n)) + \cos(3n^2)$

$= x[n] \Longrightarrow$ even

(b) $x[n]= n\,u(n)$

$x[-n] = -n\,u(-n) \neq x[n] \Rightarrow$ not even

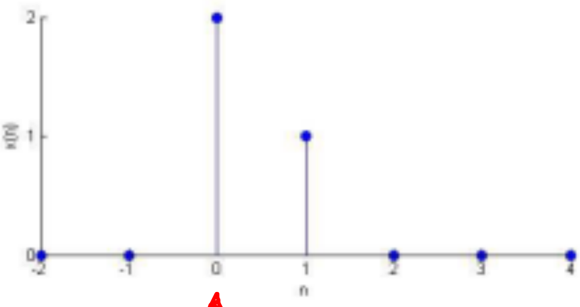
$\neq -x[n] \Rightarrow$ not odd

Not even or odd

Q3. For the shown discrete signal x(n), sketch each of the following signals [5 Points Each]:

(a) $y_1(n)= x(n/2-1)$

(b) $y_2(n)= 2\,x(-2\,n+1)$



Notice zero is here

a)

$n = 0 \longrightarrow y_1(0) = x(-1)$

$n = 1 \longrightarrow y_1(1) = x(-1)$

$n = 2 \longrightarrow y_1(2) = x(0)$

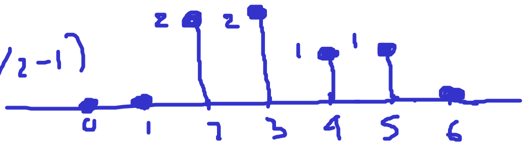
$n = 3 \longrightarrow y_1(3) = x(0)$

$n = 4 \longrightarrow y_1(4) = x(1)$

$n = 5 \longrightarrow y_1(5) = x(1)$

$n = 6 \longrightarrow y_1(6) = x(2)$

$y(n) = x(n/2-1)$



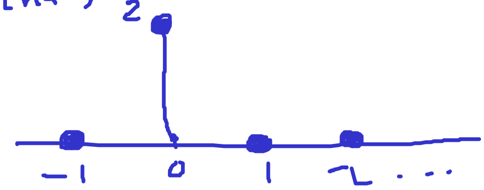
b) $y_2(n) = 2\;x(-2n+1)$

$n = 0 \longrightarrow y_2(0) = 2\;x(1)$

$n = 1 \longrightarrow y_2(1) = 2\;x(-1)$

$n = -1 \longrightarrow y_2(-1) = 2\;x(3)$

$y_2(n) = 2x(-2n+1)$



Q4. For each system, determine whether it is (i) linear, (ii) time invariant and (iii) causal [6 Points Each]:

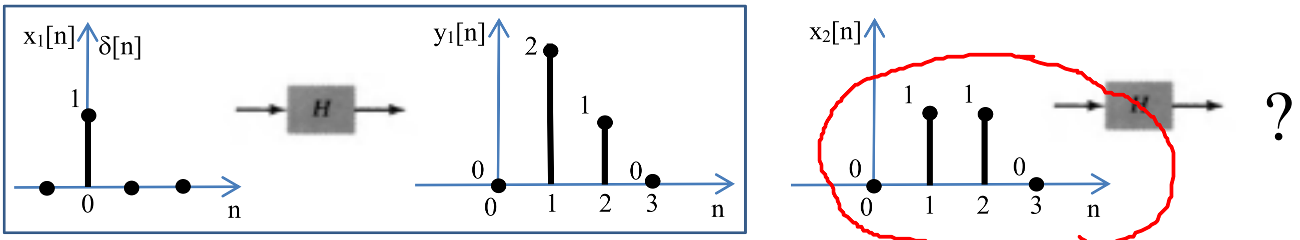
(a) $y(n) = \sum_{k=1}^{3} k \cdot y(n-k) + \sum_{m=0}^{4} x(n-m)$, $y(-1) = y(-2) = y(-3) = 0$

- This looks like a recursive system equation with zero initial conditions
  $\Longrightarrow$ Linear and Time Invariant
- Since $y(n)$ depends only on present & past values of input $x(n)$ ($x(n), x(n-1), x(n-2), x(n-3), x(n-4)$)
  $\Longrightarrow$ Causal

(b) $y(n) = \underbrace{x(n) - \frac{1}{2}x(n-1) + n \cdot x(n-2)}_{\text{Linear terms}} + 1$

- offset } nonlinear system
- output for input $x(n-n_0) = x(n-n_0) - \frac{1}{2}x(n-n_0-1) + n\,x(n-n_0-2) + 1$
- $y(n-n_0) = x(n-n_0) - \frac{1}{2}x(n-n_0-1) + (n-n_0)\,x(n-n_0-2) + 1$
  $\rightarrow$ not equal $\Longrightarrow$ Time varying
- Since $y(n)$ depends only on past & present values of $x(n)$ $\Longrightarrow$ Causal

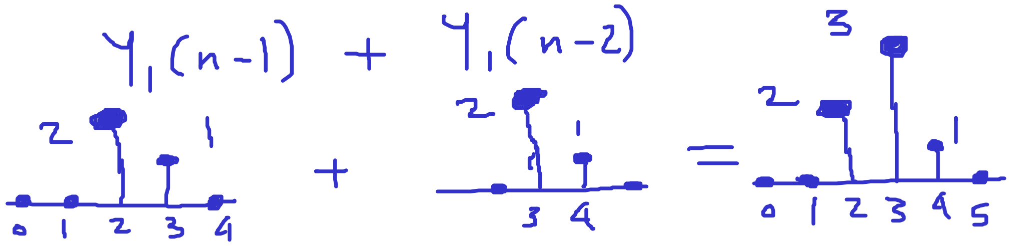
Q5. For a discrete linear time invariant system, if the output of the system $y_1[n]$ is known for a particular input $x_1[n]$ (unit sample function $\delta[n]$) as shown, compute the output of the same system for an input $x_2[n]$ shown. [10 Points]



$$x_2(n) = x_1(n-1) + x_1(n-2)$$

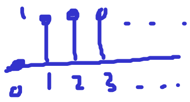
Since system is LTI,

$$y_2(n) = y_1(n-1) + y_1(n-2)$$



Q6. Determine the output of the system described by the following difference equation with input and initial conditions as specified: [5 Points]

$y[n]= x[n] + x[n-1] - y[n-1]$, $x[n]= u[n-1]$, $y[-1]= 1$



$n=0: \quad y(0) = \cancel{x(0)}^{0} + \cancel{x(-1)}^{0} - y(-1) = -1$

$n=1: \quad y(1) = x(1) + \cancel{x(0)}^{0} - y(0) = 1-(-1) = 2$

$n=2: \quad y(2) = x(2) + x(1) - y(1) = 1+1-2 = 0$

$n=3: \quad y(3) = x(3) + x(2) - y(2) = 1+1-0 = 2$

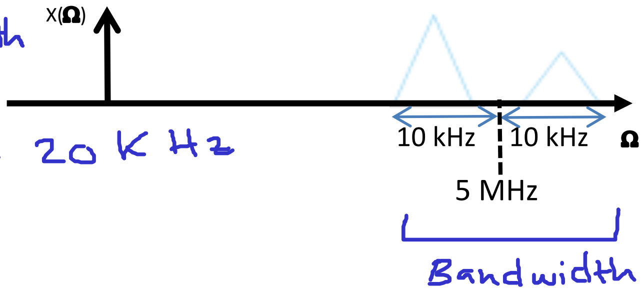
$\vdots$

Q7. Determine a suitable sampling frequency for the following signals and explain: [5 Points Each]

(a) The signal with the shown Fourier spectrum

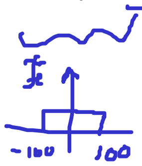
Min. Sampling freq = Signal bandwidth

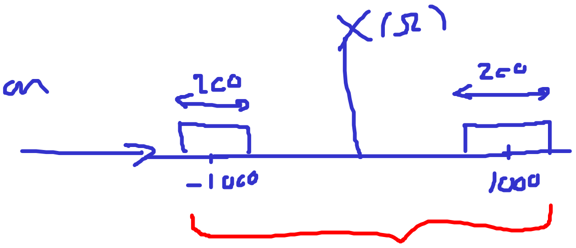
= 2 x 10 KHz = 20 KHz



(b) $x(t) = \frac{sin(100t)}{t} \bullet \cos(1000\,t)$

modulation





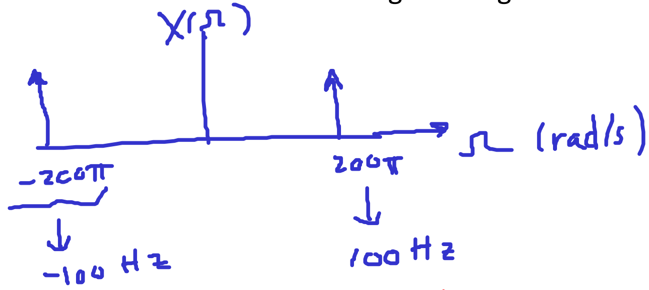
Bandwidth = 1000x2+200
= 2200 Hz

⇒ min Sampling frequency = 2200 Sa/s

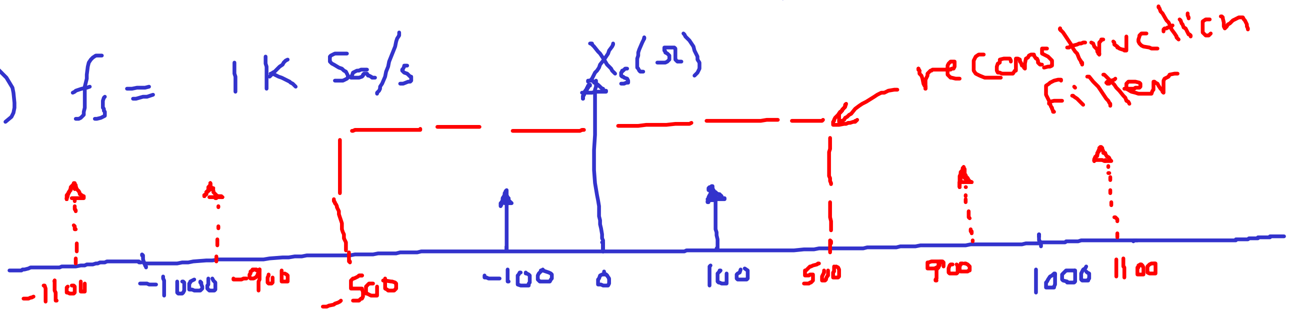
Q8. A signal $(t) = \cos(200\pi t)$ was sampled with an ideal pulse train. Sketch the continuous-time Fourier transformation for the following values of the sampling rate and estimate the reconstructed signal using a lowpass filter with cutoff frequency of $\Omega_s/2$: [10 Points]

(a) f_s = 1k Samples/s

(b) f_s= 100 Samples/s

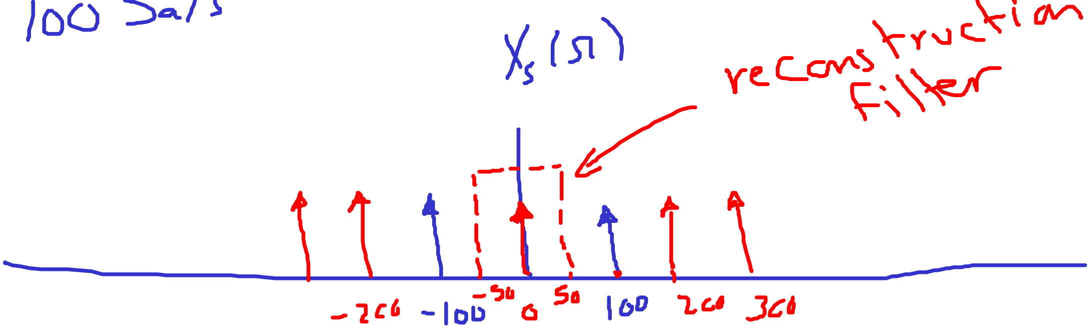


a) f_s = 1 K Sa/s



⟹ perfect reconstruction is obtained (same original signal)

b) f_s = 100 Sa/s



⟹ output of reconstruction Filter is DC ≠ original spectrum

Q9. Write correct Matlab statements to perform each of the following tasks: [5 Points Each]

(a) Generate and plot the continuous time function x(t) = e^{-t} cos (2π t/3) (u(t) – u(t-1))

```
syms t
x = exp(-t) * cos(2 * π * t/3) *(heaviside(t) - heaviside(t-1))
ezplot(x)
```

(b) Plot the discrete-time function: y[n] = cos(2π n/8)

```
n = -10:10
y = cos(2 * π *n /8)
stem (n, y)
```

(c) Compute and plot the continuous Fourier transform of function: f(t) = exp(j 2π t/3) u(t-2)

```
syms t
f = exp(j * 2 * π * t/3) * heaviside(t -2)
F = fourier (f)
```

(d) Given a 32-point discrete signal array **h**, obtain its 128-point discrete Fourier transform **H**.

```
H = fft(h, 128)
```

(e) Given two discrete signals `s1=[1 1 0 0]` and `s2=[1 1 1 1]`, compute their linear convolution.

```
S1 = fft(s1, 8)
S2 = fft(s2, 8)
S1S2 = S1 .* S2
LinConv = ifft(S1S2)
```