HW #2 Solutions

1 (d) $y(t) = 2x(-t+3)\,u(t)$

Scaling: response to $\alpha x = 2\alpha\, x(-t+3)\,u(t)$

$= \alpha\left(2x(-t+3)\,u(t)\right) = \alpha y(t)$ ✓

Additivity: $y_1 = 2x_1(-t+3)\,u(t)$

$y_2 = 2x_2(-t+3)\,u(t)$

response to $(x_1+x_2) = 2\left[x_1(-t+3) + x_2(-t+3)\right]u(t)$

$= 2x_1(-t+3)\,u(t) + 2x_2(-t+3)\,u(t)$

$= y_1 + y_2 \Rightarrow$ ✓

Linear

response to $x(t-\tau) \Rightarrow x(-(t-\tau)+3)\,u(t) \neq y(t-\tau)$

since $y(t-\tau) = x(-(t-\tau)+3)\,u(t-\tau) \Rightarrow$ time varying

Causality: since $y(1) = x(-1+3)\,u(t) = x(2)$

Then $y(t)$ depends on future values of x

$\Rightarrow$ not Causal

BIBO: since $|y| = |x|$, then it is BIBO stable

1 (f) - Linear : all linear terms : x, $\frac{d^2x}{dt^2}$

- Time invariant : only if initial Conditions are assumed to be zero
- Causal : since all output values depend on present values of x and its derivatives
- BIBO : since $|y| \leq \underbrace{\left|\frac{d^2x}{dt^2}\right| + 2|x|}_{\text{Both Finite}}$

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2 Left system

Since $x_3 = x_1 - x_2$ while $y_3 \neq y_1 - y_2$
Then system is not Linear (additivity not working)

Since $x_2 = -x_1(t-2)$ and $y_2 = -y_1(t-2)$
Then system is time-invariant

Since all outputs are zero when inputs are zero
Then system is Causal

2 Right system

since $x_3 = \frac{1}{2}x_2 - x_1$ and $y_3 = \frac{1}{2}y_2 - y_1$
Then system is Linear

Since $x_4(t) = x_2(t-1)$ while $y_4 \neq y_2(t-1)$
Then system is Time varying

Since outputs y_1 & y_3 start while inputs are still zero, Then system is not Causal

(3) Since system is LTI, we can compute output to $x(t)$ in terms of output to $x_1(t)$

Since $x(t) = x_1(t) + 2\,x_1(t-1)$

Then $y(t) = y_1(t) + 2\,y_1(t-1)$