Signals and Systems – Chapter 1

Continuous-Time Signals

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Overview of Chapter 1

- Mathematical representation of signals
- Classification of signals
- Signal manipulation
- Basic signal representation

Introduction

- Learning how to represent signals in analog as well as in digital forms and how to model and design systems capable of dealing with different types of signals
- Most signals come in analog form
- Trend has been toward digital representation and processing of data
  - Computer capabilities increase continuously

Analog vs. Discrete Signals

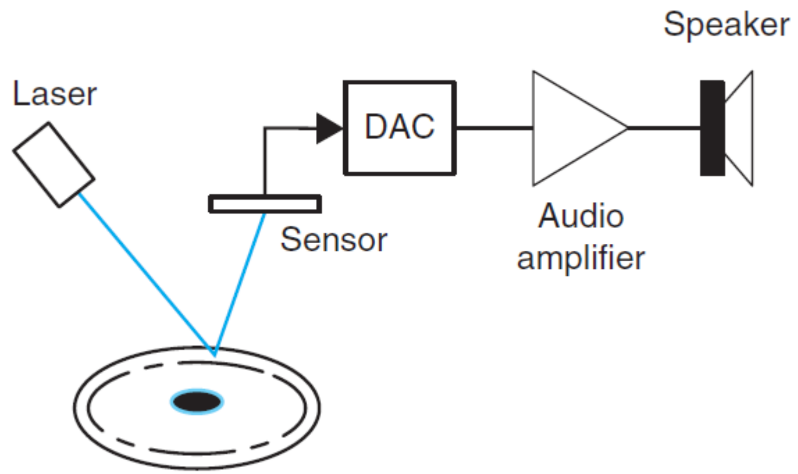
- Analog: Infinitesimal calculus (or just calculus)
  - Functions of continuous variables
  - Derivative
  - Integral
  - Differential equations
- Discrete: Finite calculus
  - Sequences
  - Difference
  - Summation
  - Difference equations

Real Life

Computer

Example of Signal Processing Application

- Compact-Disc (CD) Player
  - Analog sound signals
  - Sampled and stored in digital form
  - Read as digital and converted back to analog
  - High fidelity (Hi-Fi)



Classification of Time-Dependent Signals

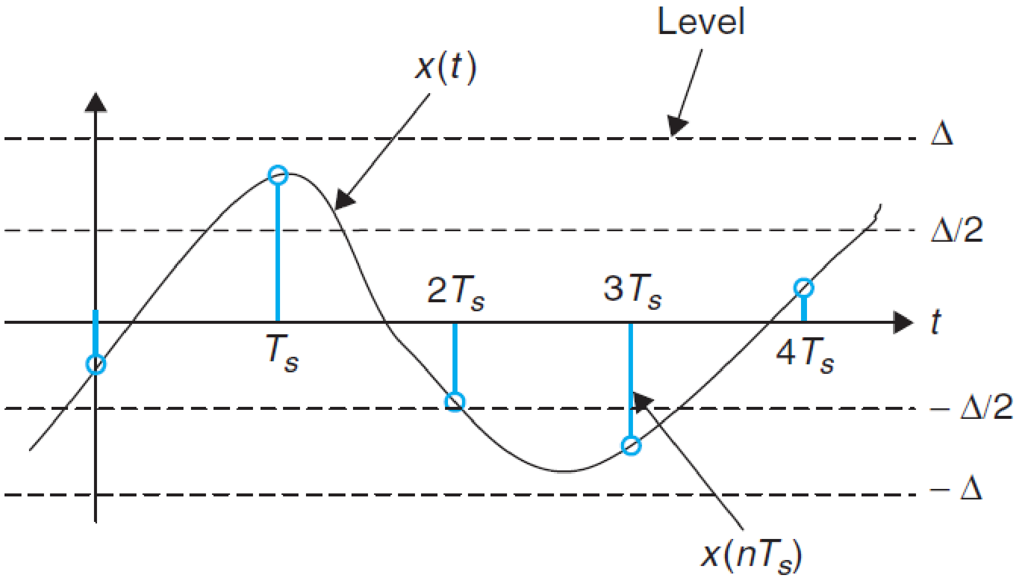
- Predictability of their behavior
  - Signals can be random or deterministic
- Variation of their time variable and their amplitude
  - Signals can be either continuous-time or discrete-time
  - Signals can be either analog or discrete amplitude, or digital
- Energy content
  - Signals can be characterized as finite- or infinite-energy signals
- Exhibition of repetitive behavior
  - Signals can be periodic or aperiodic
- Symmetry with respect to the time origin
  - Signals can be even or odd
- Dimension of their support
  - Signals can be of finite or of infinite support. Support

Continuous-Time Signals

- Continuous-amplitude, continuous-time signals are called *analog signals*
- Continuous-amplitude, discrete-time signal is called a *discrete-time signal*
- Discrete-amplitude, discrete-time signal is called a *digital signal*
- If samples of a digital signal are given as binary values, signal is called a *binary signal*

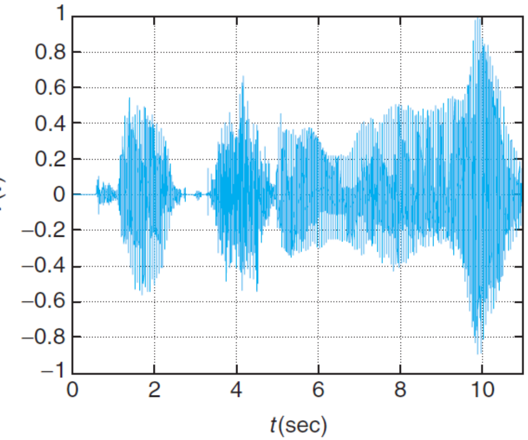
Continuous-Time Signals

- Conversion from continuous to discrete time: *Sampling*
- Conversion from continuous to discrete amplitude: *Quantization* or *Coding*



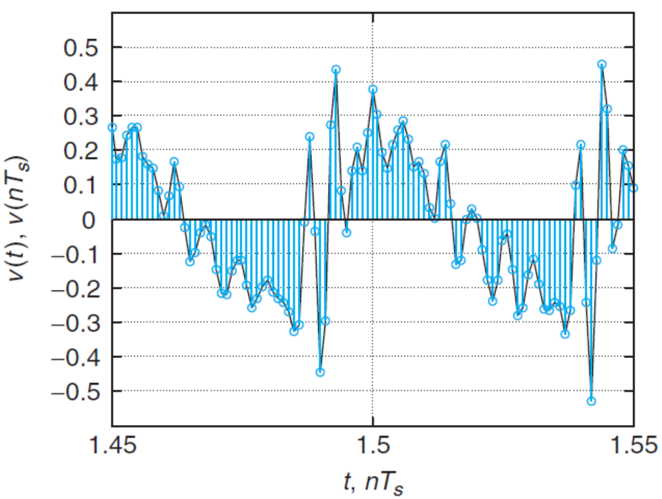
Continuous-Time Signals

- Example: Speech Signal

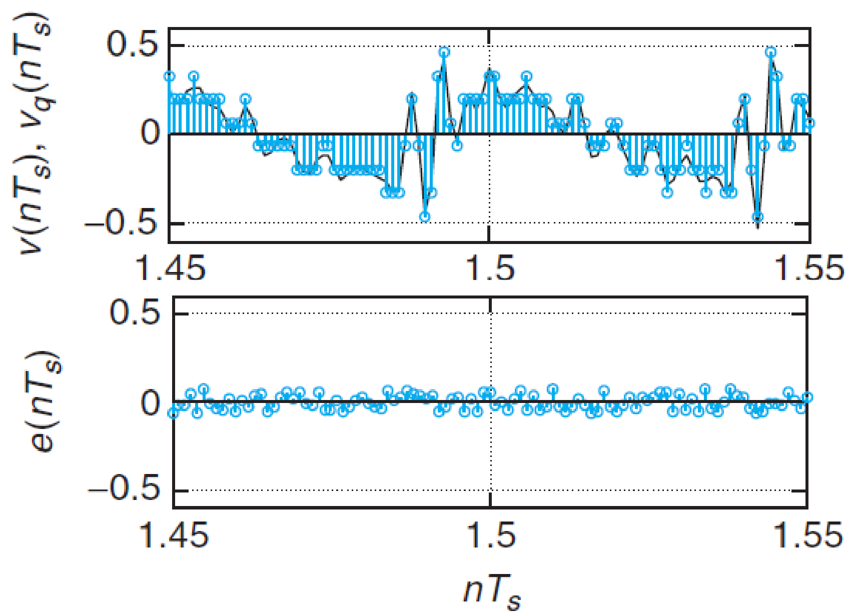


Analog Signal

Sampling



Quantization



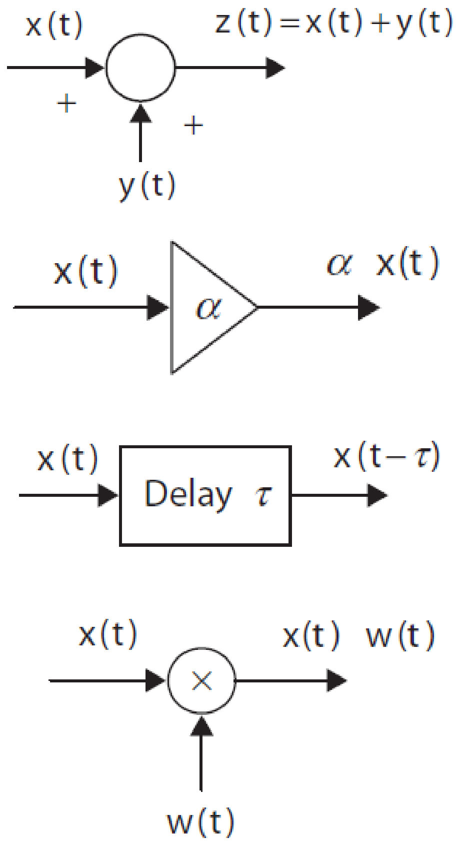
Error

Continuous-Time Signals: Examples

- Example 1: $x(t) = \sqrt{2}\cos(\pi t/2 + \pi/4) \qquad -\infty < t < \infty$
  - Deterministic, analog, periodic, odd, infinite support/energy
- Example 2: $y(t) = (1 + j)e^{j\pi t/2} \qquad 0 \le t \le 10$
  - Deterministic, analog, finite support
- Example 3: $p(t) = 1 \qquad 0 \le t \le 10$
  - Deterministic, analog, finite support

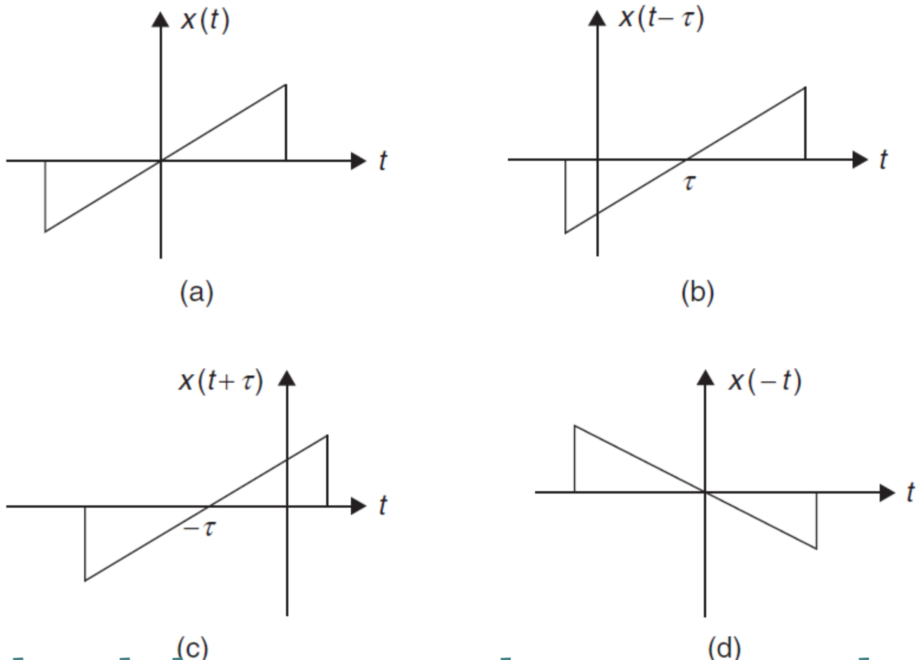
Basic Signal Operations

- Signal addition
- Constant multiplication
- Time and frequency shifting
  - Shift in time: *Delay*
  - Shift in frequency: *Modulation*
- Time scaling
  - Example: $x(-t)$ is a "reflection" of $x(t)$
- Time windowing
  - Multiplication by a window signal $w(t)$



Basic Signal Operations

- Example:
  - (a) original signal
  - (b) delayed version
  - (c) advanced version
  - (d) Reflected version



- Remark:
  - Whenever we combine the delaying or advancing with reflection, delaying and advancing are swapped
  - Ex 1: $x(-t+1)$ is reflected and delayed
  - Ex 2: $x(-t-1)$ is reflected and advanced

Basic Signal Operations

- Example: Find mathematical expressions for $x(t)$ delayed by 2, advanced by 2, and reflected when:

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

  - For delay by 2, replace t by t-2

$$x(t-2) = \begin{cases} 1 & 0 \le t-2 \le 1 \ \text{ or } \ 2 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

  - For advance by 2

$$x(t+2) = \begin{cases} 1 & 0 \le t+2 \le 1 \ \text{ or } \ -2 \le t \le -1 \\ 0 & \text{otherwise} \end{cases}$$

  - For reflection

$$x(-t) = \begin{cases} 1 & 0 \le -t \le 1 \ \text{ or } \ -1 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$$

Even and Odd Signals

- Symmetry with respect to the origin

$$x(t) \quad \text{even}: \quad x(t) = x(-t)$$

$$x(t) \quad \text{odd}: \quad x(t) = -x(-t)$$

- Decomposition of any signal as even/odd parts

$$y(t) = y_e(t) + y_o(t)$$

$$y_e(t) = 0.5\,[y(t) + y(-t)]$$

$$y_o(t) = 0.5\,[y(t) - y(-t)]$$

- Example: $x(t) = \cos(2\pi t + \theta) \qquad -\infty < t < \infty$
  - Neither even nor odd for $\theta \neq 0$ or multiples of $\pi/2$

Periodic and Aperiodic Signals

- Analog signal $x(t)$ is periodic if
  - It is defined for all possible values of t, $-\infty<t<\infty$
  - there is a positive real value T_0, called the period, such that for some integer k, $x(t+kT_o) = x(t)$
- The period is the smallest possible value of $T_0>0$ that makes the periodicity possible.
  - Although NT_0 for an integer $N>1$ is also a period of $x(t)$, it should not be considered *the* period
    - Example: $\cos(2\pi\, t)$ has a period of 1 not 2 or 3

Periodic and Aperiodic Signals

- Analog sinusoids of frequency $\Omega_0>0$ are periodic of period T_0 $2\pi/\Omega_0$.
  - If $\Omega_0=0$, the period is not well defined.
- The sum of two periodic signals $x(t)$ and $y(t)$, of periods $T1$ and $T2$, is periodic if the ratio of the periods $T1/T2$ is a rational number N/M, with N and M being nondivisible.
  - The period of the sum is $MT1=NT2$
- The product of two periodic signals is not necessarily periodic
  - The product of two sinusoids is periodic.

Periodic and Aperiodic Signals

- Example 1

Consider a periodic signal $x(t)$ of period T_0. Determine whether the following signals are periodic, and if so, find their corresponding periods:

(a) $y(t) = A + x(t)$.

(b) $z(t) = x(t) + v(t)$ where $v(t)$ is periodic of period $T_1 = NT_0$, where N is a positive integer.

(c) $w(t) = x(t) + u(t)$ where $u(t)$ is periodic of period T_1, not necessarily a multiple of T_0. Determine under what conditions $w(t)$ could be periodic.

- Example 2

Let $x(t) = e^{j2t}$ and $y(t) = e^{j\pi t}$, and consider their sum $z(t) = x(t) + y(t)$, and their product $w(t) = x(t)y(t)$. Determine if $z(t)$ and $w(t)$ are periodic, and if so, find their periods. Is $p(t) = (1 + x(t))(1 + y(t))$ periodic?

Finite-Energy and Finite-Power Signals

- Concepts of energy and power introduced in circuit theory can be extended to any signal
  - Instantaneous power
  - Energy
  - Power

$$\boxed{p(t) = v(t)i(t) = i^2(t) = v^2(t)}$$

$$\boxed{E_T = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} i^2(t)dt = \int_{t_0}^{t_1} v^2(t)dt}$$

$$\boxed{P_T = \frac{E_T}{T} = \frac{1}{T}\int_{t_0}^{t_1} i^2(t)dt = \frac{1}{T}\int_{t_0}^{t_1} v^2(t)dt}$$

Finite-Energy and Finite Power Signals

- Energy of an analog signal $x(t)$

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

- Power of an analog signal $x(t)$

$$\boxed{P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt}$$

- Signal is *finite energy* (or *square integrable*) if $E_x < \infty$
- Signal is *finite power* if $P_x < \infty$

Finite-Energy and Finite Power Signals: Example

Find the energy and the power of the following:

(a) The periodic signal $x(t) = \cos(\pi t/2 + \pi/4)$.
(b) The complex signal $y(t) = (1+j)e^{j\pi t/2}$, for $0 \leq t \leq 10$ and zero otherwise.
(c) The pulse $z(t) = 1$, for $0 \leq t \leq 10$ and zero otherwise.

$$E_x = \int_{-\infty}^{\infty} \cos^2(\pi t/2 + \pi/4)dt \to \infty$$

$$P_x = \frac{1}{8}\int_{0}^{4} \cos(\pi t + \pi/2)dt + \frac{1}{8}\int_{0}^{4} dt = 0 + 0.5 = 0.5$$

$$E_y = \int_{0}^{10} |(1+j)e^{j\pi t/2}|^2 dt = 2\int_{0}^{10} dt = 20$$

Finite Energy Signals: Zero Power

$$E_z = \int_{0}^{10} dt = 10$$

$$P_x = \lim_{T\to\infty} \frac{E_x}{2T} = 0$$

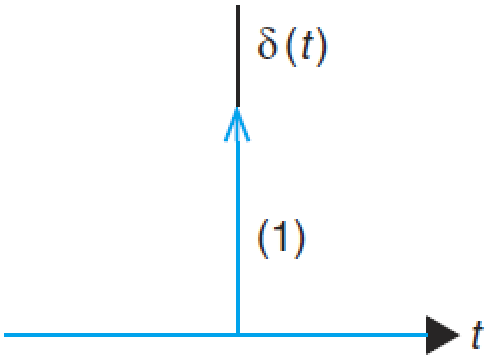
Representation Using Basic Signals

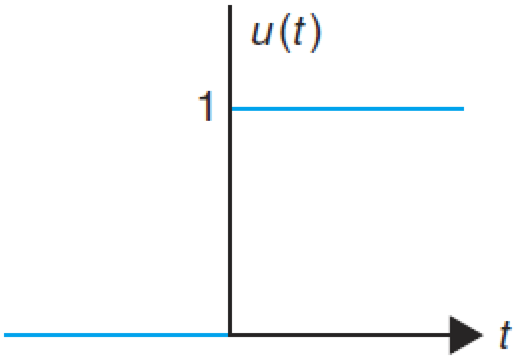
- A fundamental idea in signal processing is to attempt to represent signals in terms of basic signals, which we know how to process
  - Impulse
  - Unit-step
  - Ramp
  - Sinusoids
  - Complex exponentials

Impulse and Unit-Step Signals

- The impulse signal $\delta(t)$ is:
  - Zero everywhere except at the origin where its value is not well defined
  - Its area is equal to unity
- Impulse signal $\delta(t)$ and unit step signal u(t) are related by:

$$\boxed{\begin{aligned} u(t) &= \int_{-\infty}^{t} \delta(\tau)d\tau \\ \delta(t) &= \frac{du(t)}{dt} \end{aligned}}$$





Ramp Signal

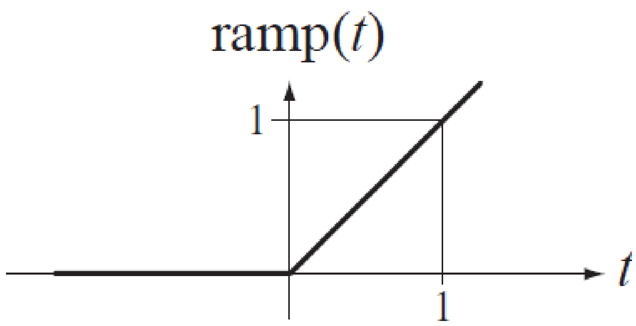
The ramp signal is defined as

$$r(t) = t\, u(t)$$

Its relation to the unit-step and the unit-impulse signals is

$$\frac{dr(t)}{dt} = u(t)$$

$$\frac{d^2 r(t)}{dt^2} = \delta(t)$$

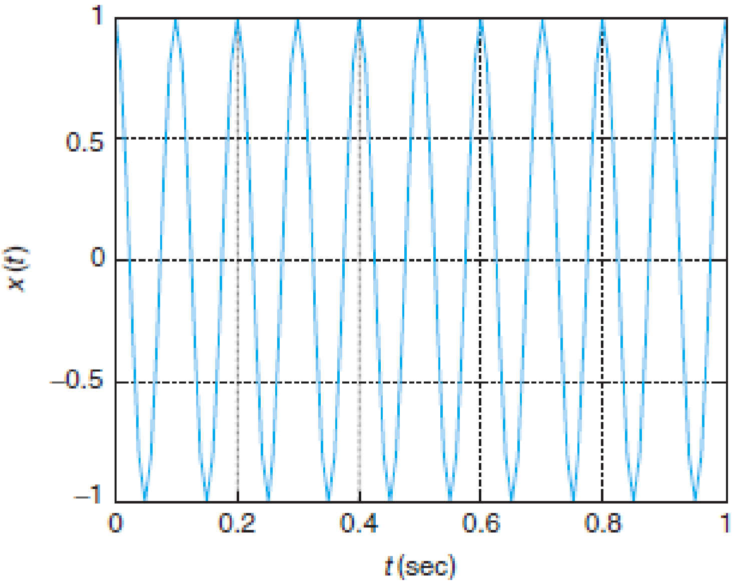


Sinusoids

Sinusoids are of the general form

$$A\cos(\Omega_0 t + \theta) = A\sin(\Omega_0 t + \theta + \pi/2) \qquad -\infty < t < \infty$$

$$\Omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

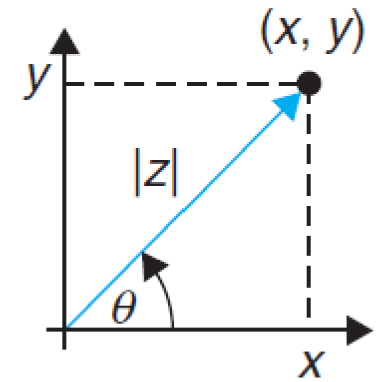


Review of Complex Numbers

- A complex number z represents any point (x, y):

$$z = x + j\,y,$$

  - x =Re[z] (real part of z)
  - y =Im[z] (imaginary part of z)
  - j =Sqrt(-1)



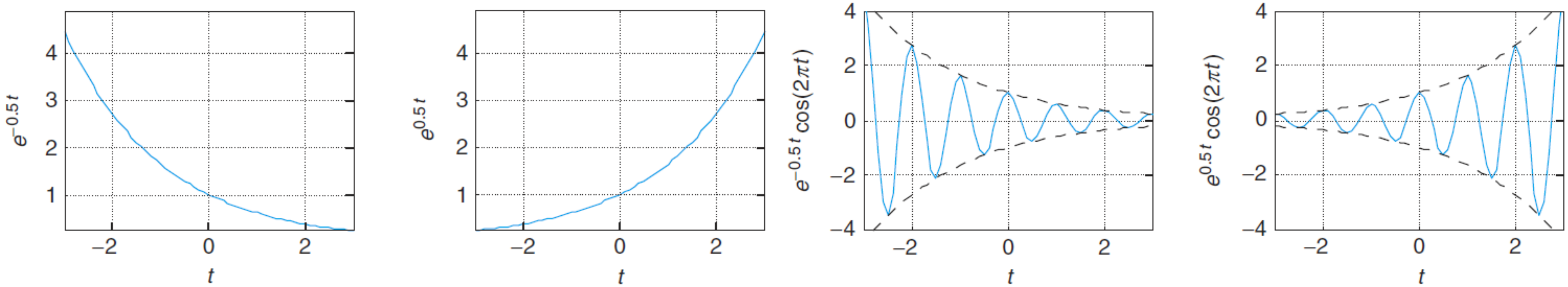
- Mathematical representations:
  - Rectangular or polar form $z = x + jy = |z|e^{j\theta}$
  - Magnitude $|\vec{z}| = \sqrt{x^2 + y^2} = |z|$ and Phase $\theta = \angle\vec{z} = \angle z$
- Conjugate $\boxed{z^* = x - jy = |z|e^{-j\angle z}}$

Complex Exponentials

A complex exponential is a signal of the form

$$x(t) = Ae^{at}$$
$$= |A|e^{rt}\left[\cos(\Omega_0 t + \theta) + j\sin(\Omega_0 t + \theta)\right] \quad -\infty < t < \infty$$

where $A = |A|e^{j\theta}$, and $a = r + j\Omega_0$ are complex numbers.



- Depending on the values of *A* and *a*, several signals can be obtained from the complex exponential

Basic Signal Operations—Time Scaling, Frequency Shifting, and Windowing

Given a signal $x(t)$, and real values $\alpha \neq 0$ or 1, and $\phi > 0$:

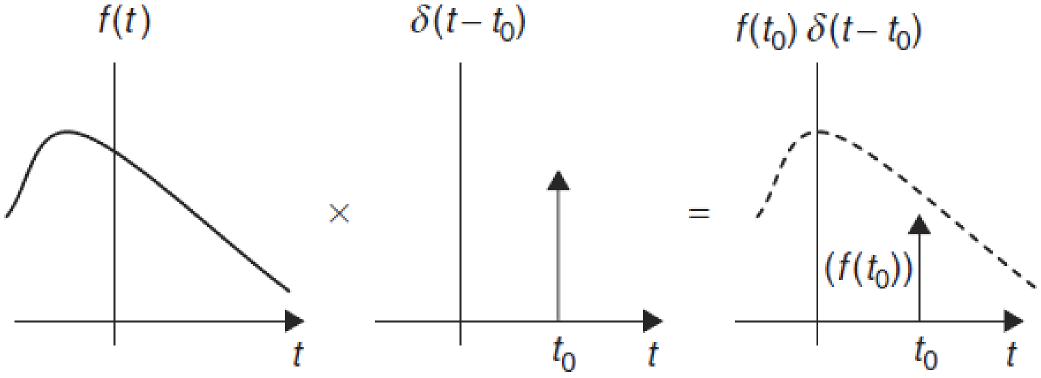
- $x(\alpha t)$ is said to be contracted if $|\alpha| > 1$, and if $\alpha < 0$ it is also reflected.
- $x(\alpha t)$ is said to be expanded if $|\alpha| < 1$, and if $\alpha < 0$ it is also reflected.
- $x(t)e^{j\phi t}$ is said to be shifted in frequency by ϕ radians.
- For a window signal $w(t)$, $x(t)w(t)$ displays $x(t)$ within the support of $w(t)$.

Sifting Property

- Property of the impulse function

$$\int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)dt = f(\tau)\int_{-\infty}^{\infty} \delta(t-\tau)dt$$

$$= f(\tau) \qquad \text{for any } \tau$$



Problem Assignments

- Problems: 1.4, 1.5, 1.12, 1.13, 1.14, 1.18
- Partial Solutions available from the student section of the textbook web site