NONLINEAR DYNAMICAL MODELING OF ECG SIGNALS BASED ON PHASE DENSITY MATRIX REPRESENTATION

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Abstract–**Four types of ventricular arrhythmias (PVC, VB, VT and VF) were considered; a new matrix, Phase space density matrix, was generated from the reconstructed state space of the ECG signal, and a number of features were extracted from its contents to be used in the classification process, Three statistical classifiers are used in the classification, results confirmed the robustness of the new technique and demonstrate its value as a diagnostic tool, where sensitivity was 96.15%, 76.92%, 84.62%, and 100% for PVC, VT, VB, VF respectively.**

Keywords – **ECG,arrhythmia analysis,nonlinear dynamics**

I. INTRODUCTION

Cardiac arrhythmias are considered among the most fatal conditions of the human physiology [1]. In many cases when such abnormalities are correctly diagnosed early enough, it is possible to find ways to treat the patient prevent further complications of the case. As a result, the detection and classification of such conditions have received a great deal of research work since the introduction of electrocardiography and later of computerized diagnostic system [2]. In particular, it is of prime importance to differentiate potentially lethal arrhythmias such as ventricular fibrillation (VF) and ventricular tachycardia (VT), from more benign problems as manifested in superventricular tachycardia (SVT) [3]. Therefore, the development of accurate noninvasive techniques for assessing the risk of lethal arrhythmias is essential to reducing mortality from cardiac deaths.

Various detection algorithms have been reported, which can be classified as linear techniques such as sequential hypothesis testing[4], Autoregressive Modeling of ECG signal[5], frequency domain features[6], wavelet analysis[7],[8], and nonlinear techniques, which uses the concept of chaotic systems to describe and extract some features from the ECG signal [14]. All these methods exhibit advantages and disadvantages, some being too difficult to implement and compute for AEDs and ICDs, and some having low specificity in differentiating between various types of arrhythmias. In this paper we proposed a novel technique to be used in arrhythmia classification, which take into consideration the problem of characterizing the nonlinear dynamics (chaos theory) of the ECG signal and its variation with different arrhythmia types, where a new matrix, *Phase Space Density Matrix*, was reconstructed from the phase space trajectory of the ECG signals and some features was extracted from this matrix to be used in arrhythmia detection and classification. The proposed implementations were used to compute these features for a large number of independent ECG signals belonging to five different ECG signal types from the MIT-BIH arrhythmia Database [9]. The results are studied to detect statistically significant differences among different arrhythmia types. Finally, statistical classification techniques are used to assess the possibility

II. PHASE DENSITY MATRIX

In this method a two-dimensional phase space trajectory of the ECG signal was reconstructed using the time delay method, where the delay time τ was calculated from the first minimum of the average mutual information function [10]. The resulting phase space can be obtained by using the ECG signal $x(k)$ and its delayed version $x(k+\tau)$:

$$\text{Phase Space} = \begin{bmatrix} x(1) & x(2) & . \; . \; . & x(M) \\ x(1+t) & x(2+t) & . \; . \; . & x(M+t) \end{bmatrix}. \quad (1)$$

The phase space plot for two dimension phase space is obtained by plotting row 2 of the reconstructed matrix against row1. The values in the matrix are normalized between 0 and 1, and then the phase space area is divided into small square areas of equal size such that the phase space is divided into a grid of N x N squares. A phase space density matrix C is now obtained with its elements C(i, j) equal to the number of phase space points falling in a grid. The points in the phase space were restricted to a specific region of the phase space for each type of arrhythmia. The phase space plots and the corresponding phase space density plots for the NSR, PVC, VT, VB and VF signals are shown in Fig. 1 and Fig. 2 respectively. It is observed that the distribution of phase space points for NSR and different types of arrhythmias were clearly different as shown in Fig.2.

If we divide the phase space plots of the ECG signal into a grid of 20x20 squares, then we will have 400 elements that is too much to be used as a features vector in the classification process, so a reduced number of features can be calculated based on the co-occurrence matrix, a method used for the purpose of texture discrimination [11], some of these features are defined by the equations that follow, where μ_x, μ_y and σ_x, σ_y denote the mean and standard deviations of the row and column sums of the density matrix, respectively.

a. The contrast (CON)

$$CON = \sum_{i,j=1}^{N} (i-j)^2 . C(i,j) \quad (2)$$

b. The Angular Second Moment (ASM)

$$ASM = \sum_{i,j=1}^{N} (C(i,j))^2 \quad (3)$$

c. The entropy (ENT)

$$ENT = -\sum_{i,j=1}^{N} C(i,j) . \log(C(i,j)) \quad (4)$$

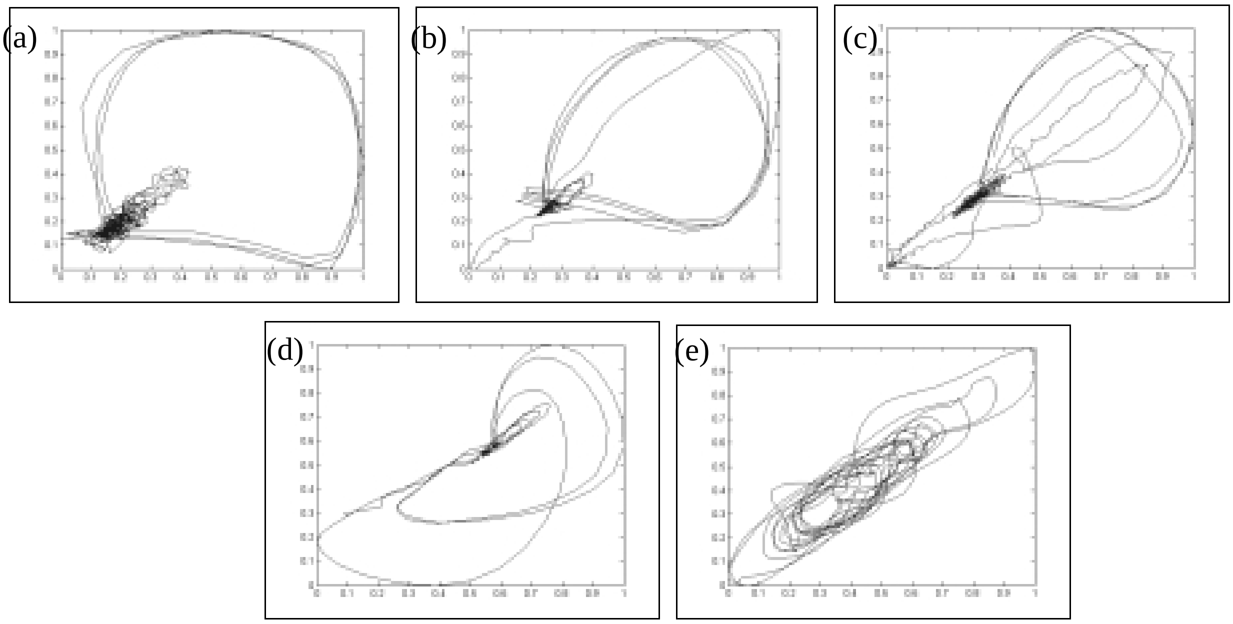

Figure 1 The phase space plots of (a) NSR, (b) PVC, (c) VB, (d) VT and (e) VF signals

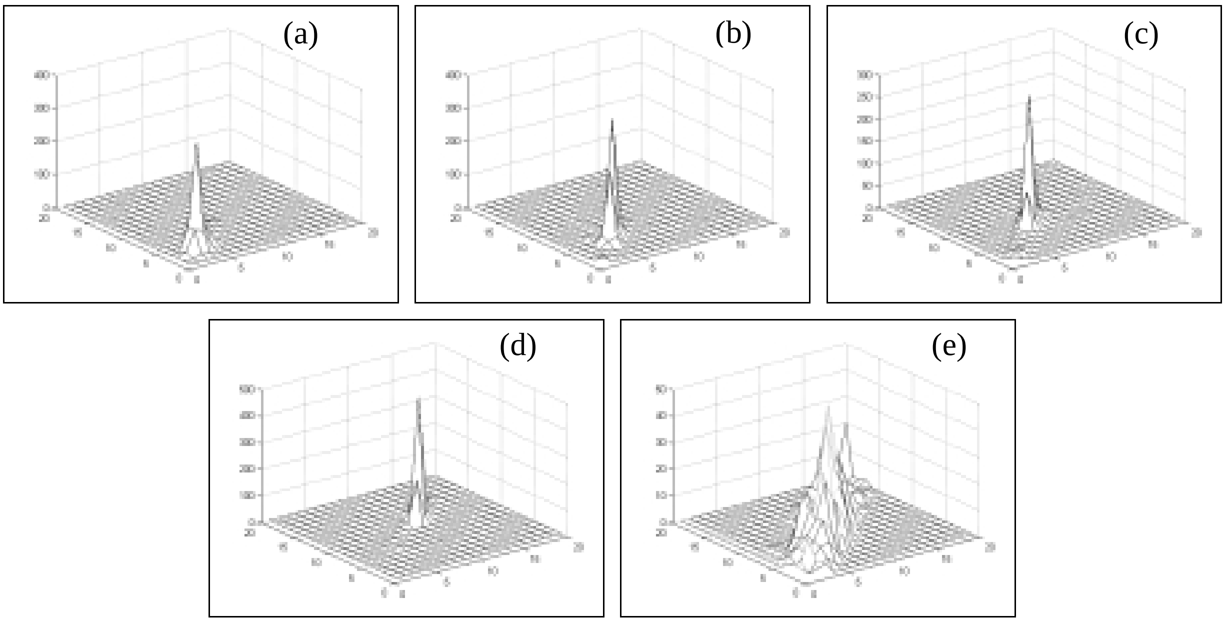

Figure 2 Phase space density plots of(a) NSR, (b) PVC, (c) VB, (d) VT and (e) VF signals

d. The Correlation (COR)

$$COR = \frac{\sum_{i,j=1}^{N} ijC(i,j) - m_x m_y}{S_x S_y} \qquad (5)$$

e. Maximum density:

$$MAX(C(i,j)) \qquad (6)$$

f. Inverse difference moment:

$$\sum_{i,j=1}^{N} \frac{C(i,j)}{1+(i-j)^2} \qquad (7)$$

III. Results and Discussion

The ECG signals used in this paper were obtained from the MIT-BIH arrhythmia database [9]; the data set was composed of five different types including normal sinus rhythm (NSR), premature ventricular couplet (PVC), ventricular bigeminy (VB), ventricular tachycardia (VT), and ventricular fibrillation (VF). The data set was divided into learning and testing data set, 52 independent signals for the learning set of each type and 26 independent signals for the test set of each type with each signal length 3 sec. All the signals were resampled at 360 samples/s.

The two dimension phase space trajectory of the ECG signals was reconstructed using the delay time embedding method, where the delay was chosen to be 5 samples which calculated from the first minimum of the mutual information function [10],[12]. Then the phase space plots of the signals were divided into a grid of 20x20 squares and the numbers of points *C(i,j)* within each square was computed to form the phase space density matrix *C.* The six features from the density matrix (CON, ASM, ENT, COR, MAX, and Inverse difference moment) were extracted to form the features vectors. I used the significance test in this paper to assess the use of the parameters extracted from the new technique for discriminating between different ECG signal types. Results of significance test for each feature were shown in tables I-VI

TABLE I. P-values of t-test for CON.

Type	PVC	VT	VB	VF
NR	<1.0e-16	<1.0e-16	<1.0e-16	<1.0e-16
PVC		<1.0e-16	0.0013	**0.7691**
VT			**0.056**	0.0002
VB				0.0092

TABLE II. P-values of t-test for ASM.

Type	PVC	VT	VB	VF
NR	<1.0e-16	<1.0e-16	0.0006	<1.0e-16
PVC		<1.0e-16	<1.0e-16	<1.0e-16
VT			0.0037	<1.0e-16
VB				<1.0e-16

TABLE III. P-values of t-test for ENT.

Type	PVC	VT	VB	VF
NR	**0.133**	<1.0e-16	0.0006	<1.0e-16
PVC		<1.0e-16	<1.0e-16	<1.0e-16
VT			<1.0e-16	<1.0e-16
VB				<1.0e-16

TABLE IV. P-values of t-test for COR.

Type	PVC	VT	VB	VF
NR	<1.0e-16	**0.8116**	**0.0735**	<1.0e-16
PVC		0.004	0.0001	<1.0e-16
VT			**0.1938**	<1.0e-16
VB				<1.0e-16

TABLE V. P-values of t-test for MAX.

Type	PVC	VT	VB	VF
NR	<1.0e-16	0.0038	**0.3664**	<1.0e-16
PVC		<1.0e-16	<1.0e-16	<1.0e-16
VT			**0.051**	<1.0e-16
VB				<1.0e-16

TABLE VI. P-values of t-test for Inverse difference moment.

Type	PVC	VT	VB	VF
NR	0.0129	<1.0e-16	0.0062	<1.0e-16
PVC		<1.0e-16	<1.0e-16	<1.0e-16
VT			<1.0e-16	<1.0e-16
VB				<1.0e-16

As indicated from Tables I-VI, the results confirm that normal ECG signals can be statistically differentiated from abnormal by using the extracted features of the phase space density matrix. The very low p-values suggest the rejection of the null hypothesis and hence the presence of a significant difference. For example, when using ASM and the Inverse difference moment, there is significant difference between all pairs of arrhythmia types at 5% level, but when using CON, there is significant difference between all pairs at 5% level except between VT and VB which are significant at the 10% level, and between PVC and VF which are no statistically significant difference between them (shown in boldface inside the table). Also the other features show a very significant difference between some arrhythmia types and no statistically significant difference between other types. So we can merge all of these features in one vector to be used in the detection and classification process of different arrhythmias types. three statistical classifiers are used in this paper; minimum distance classifier, Bayes minimum-error classifier, and voting k-nearest neighbor (k-NN) classifier [13][14].

In the classification process we first tried to use each feature separately but the result was not be good, but when we combined all the features extracted from the proposed technique the results showed the robustness of the new technique in the detection of abnormality of the ECG signal. Classification results for only normal versus abnormal ECG shown in Table VII, and the results of applying the three statistical classifiers to classify the 5 different ECG types are listed in Table VIII.

TABLE VII. Results of the three classifiers (Detection)

Classifier	Specificity	Sensitivity
Min. distance	50.0000	82.6923
Bayes	96.1538	99.0385
K-NN (K=2)	79.1667	95.7895

TABLE VIII. Results of the three classifiers (Classification)

	Min. Distance	Bayes	K-NN (K=2)
Specificity	50.0000	96.1538	86.3636
Sensitivity for PVC	61.5385	96.1538	77.7778
Sensitivity for VT	57.6923	76.9231	76.9231
Sensitivity for VB	23.0769	84.6154	87.5000
Sensitivity for VF	100.0000	100.000	100.000

Results of the detection and classification process showed that Bayes minimum-error classifier seems to provide the best results which means that the clusters follow a Gaussian distribution, followed by the k-nearest neighbor classifier (the best at k=2). And the results of minimum distance classifier were rather poor which means that the classes are not linearly separable.

IV. CONCLUSIONS

Applying nonlinear signal processing techniques to signals like ECG provides very useful information for detection of cardiac abnormalities. The proposed technique have been shown to be effective for the classification of cardiac arrhythmias in critically ill patients as shown in the results of a large data set of actual ECG signals from five different classes, which indicate the value of such techniques in the diagnosis of heart disease in intensive care units (ICU).

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