STUDY OF THE EFFECT OF NOISE ON THE CALCULATION OF DIFFUSION INDICES IN DIFFUSION TENSOR MAGNETIC RESONANCE IMAGING

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Abstract—**Diffusion Tensor MRI (DTI) combines the measurement of Diffusion Tensor MRI combines the measurement of the effective diffusion tensor and the structure information extraction in vivo. It is well known that Diffusion measurements are extremely sensitive to the noise combination. In this work, we will present a throughout review on different structures in the brain, quantitative noise anisotropy indices as fractional anisotropy (FA), relative anisotropy (VA) and The volume ratio (VR). We will present also the origin of noise affecting the DTI, and how can background noise affects the DTI measurement, using Monte Carlo simulation. We found that as the Signal to noise ratio (SNR) decreases, the deviation from the original values increases, so that, the isotropic structures appears anisotropic and low anisotropic structures appears more anisotropic.**

Keywords–**Diffusion Imaging, noise, Monte Carlo methods**

I. INTRODUCTION

Diffusion tensor imaging (DTI) is a non-invasive method of characterizing tissue micro-structure. In simple models of water diffusion in tissues, the directional dependence of diffusion is defined by the diffusion tensor [1]. The advantage of this modality lies in the fact that the changes in water diffusion, produced by alterations in brain biochemistry, can be observed on diffusion weighted (DW) images long before the effects of ischemic injury can be seen on conventional T_1, or T_2 weighted images [2].

Measurement of the diffusion tensor (D) within a voxel enables the mobility of water to be characterized along orthotropic axes, allows a macroscopic voxel-averaged description of fiber structure, orientation [4] and fully quantitative evaluation of the microstructural features of healthy and diseased tissue [2]. D is estimated using a set of diffusion-weighted images. Image noise produces errors in the calculated tensor and hence in its eigenvalues (principal diffusivities) [4] and qualitative measures of diffusion anisotropy. Random variations in these quantities complicate the analysis and interpretation of DTI experiments [3].

Diagonalizing D produces eigenvalues and eigenvectors, which are the effective principal diffusivities along the orthotropic axes of the medium. Using the eigenvalues of D allows the determination of quantitative measures of diffusion anisotropy which are independent of rotation and translation within the laboratory frame of reference. This rotational invariance is required if meaningful comparisons between different subjects and data originating from different centers is to be undertaken.

While diffusion tensor imaging (DTI) holds out the prospect of obtaining detailed microstructural and physiological information on isotropic and anisotropic diffusion in vivo, it is, nevertheless, exquisitely sensitive to the detrimental effects of experimental noise [1].

In this paper, we try to quantify the effects of experimental noise on the calculation of dif-fusion using Monte Carlo simulations. We examine how diffusion anisotropy indexes vary with the SNR for media exhibiting spherical (isotropic), linear, and planar diffusion. In fact the measured anisotropy of an inhomogeneous medium becomes progressively greater noise ratio (SNR) decreases.

II. PROBLEM FORMULATION

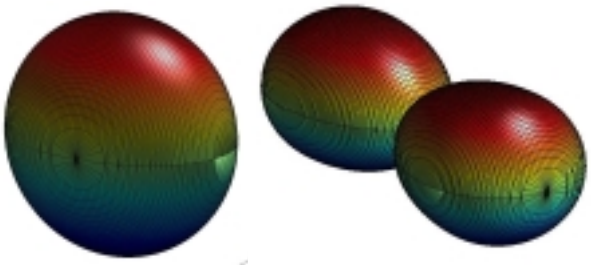
A. Diffusion profiles of different tissue types

There are three main categories into which the human brain tissue can be classified to:

1) Cerebro-spinal fluid (CSF): located around the brain and the ventricles. Its diffusion profile is much like that of unconstrained diffusing water as it is a homogeneous isotropic tissue. Its principal eigen-diffusivities (eigenvalues) would be related such that $\lambda_1 = \lambda_2 = \lambda_3$.

2) Gray matter (GM): constituting the brain cortex with its nerved cell bodies and nerve centers. The relation between its principal eigen-diffusivities is such that $\lambda_1 \approx \lambda_2 \approx \lambda_3$.

3) White matter (WM): compromising the main nerve fiber bundles in the brain. Its principal eigen diffusivities would have the typical relation of $\lambda_1 >> \lambda_2 \approx \lambda_3$. Then, the corresponding ellipsoid would be of "cigar"-shape [6] (see Fig. 1)



(a) (b)

Fig.1.Graphical illustration of a diffusion tensor (a), and its estimate based on single direction diffusion weighted imaging (b) for different directions.

B. Diffusion Anisotropy indices

Several scalar indices have been proposed to characterize diffusion anisotropy. Initially, simple indices, calculated from diffusion-weighted images or apparent diffusion coefficients (ADCs) obtained in perpendicular directions were used [7]. They are clearly dependent on the choice of directions made for the measurements. The degree of anisotropy would then vary according to the respective orientation of the gradient hardware and the tissue frames of reference and would generally be underestimated. Here again, invariant indices must be found to avoid such biases and provide an objective, intrinsic structural information [1].

Invariant indices are thus made of combinations of the terms of the diagonalized diffusion tensor, i.e., the eigenvalues λ_1, λ_2 and λ_3. The most commonly used invariant indices are the relative anisotropy (RA), the fractional anisotropy (FA), and the volume ratio (VR) indices, defined respectively as:

$$FA = \frac{\sqrt{3[(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2]}}{\sqrt{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}, \quad (1)$$

where,
$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}, \quad (2)$$

$$RA = \frac{\sqrt{[(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2]}}{\sqrt{3}\bar{\lambda}}, \quad (3)$$

$$VR = \frac{\lambda_1 \lambda_2 \lambda_3}{\bar{\lambda}^3}. \quad (4)$$

The FA measures the fraction of the magnitude of D that can be ascribed to anisotropic diffusion. The RA, a normalized standard deviation, also represents the ratio of the anisotropic part of D to its isotropic part. FA and RA vary between 0 (isotropic diffusion) and 1(=2 for RA) (infinite anisotropy). As to the VR, it represents the ratio of the ellipsoid volume to the volume of a sphere of radius equal to the average eigenvalue and its range is from 1 (isotropic diffusion) to 0 [7].

C. Background Noise

There are number of artifacts causing issues DTI. To name a few, subject contrast, eddy currents, magnetic susceptibility effect, hardware issues and image background [5]. The background gradient can destroy diffusion measurement [8], which is our main focus during the study. The image intensity in magnetic resonance magnitude images in the presence of noise are shown governed by a Rician distribution. It is common practice to assume the noise magnitude MRI images is described be a Gaussian distribution. The power of the noise is then often estimated from the standard deviation pixel signal intensity in an image region with no NMR signal [9].

Gudjartsson *et al* proved that the theoretical distribution for magnitude and phase images can be reduced to the Gaussian distribution for even fairly small SNR [9].

In common with all MRI acquisitions, background noise causes the DWI intensity to approach a baseline 'noise floor' as one progressively increases the degree of diffusion weighting. Even above this baseline, noise in DTIs can introduce significant bias in the estimates of the eigenvalues, which makes isotropic media appear anisotropic, and anisotropic media appear more anisotropic [7]. This also affects the directionality of WM.

III. METHODS

The influence of noise on the estimates of anisotropy indices was studied by Monte Carlo computer simulation, written using Matlab, at different degrees of anisotropy representing the three compartments of the brain [10-11-12]. A simulation started by assigning the eigenvalues of the diffusion tensor. For each degree of anisotropy the following simulation procedures have been performed.

The diffusion tensor in the principal coordinate was defined by assigning the eigenvalues to the diagonal elements of a 3x3 diagonal matrix (D_{diag}). Its representation, D**,** in the laboratory frame was obtained by applying the rotation matrix associated with the orientation to the diagonal matrix [4].

The six ADC values along the directions defined by the diffusion tensor imaging scheme were determined from the tensor components, D_{ij}, in the laboratory frame and the diffusion weighted signal intensities, S_0 and S_b, in 12 non-collinear magnetic field gradient directions ((1, 0, 0.5), (0, 0.5, 1), (0.5, 1, 0) ,(1, 0.5, 0), (0, 1, 0.5), (0.5, 0, 1), (1, 0, -0.5), (0, -0.5, 1), (0.5, 1, 0), (1, -0.5, 0), (0, 1, -0.5), (-0.5, 0, 1))and their corresponding to *b* value is (1000 s/mm²) were computed from the known ADC and *b* values [4]. White Gaussian distributed random noise (WGN(0,1)) was added to the ideal signal , but with modifications in the density to give the required value of the signal to noise ratio (SNR). The SNR varies from 5 t0 100 dB, with increment by 5.

Noise perturbed signal intensities were used to recalculate the noise affected ADC values [4]. The diffusion tensor components, D_{ij}, eigen-values and were recalculated. Anisotropy indices were estimated according to their respective definitions using the noise perturbed parameters of the diffusion tensor. The related mathematical details of the above procedures are given in the Appendix. At each SNR level 10000 replicate simulations were performed including the calculation of the mean of ADCs, FA, RA and VR [12].

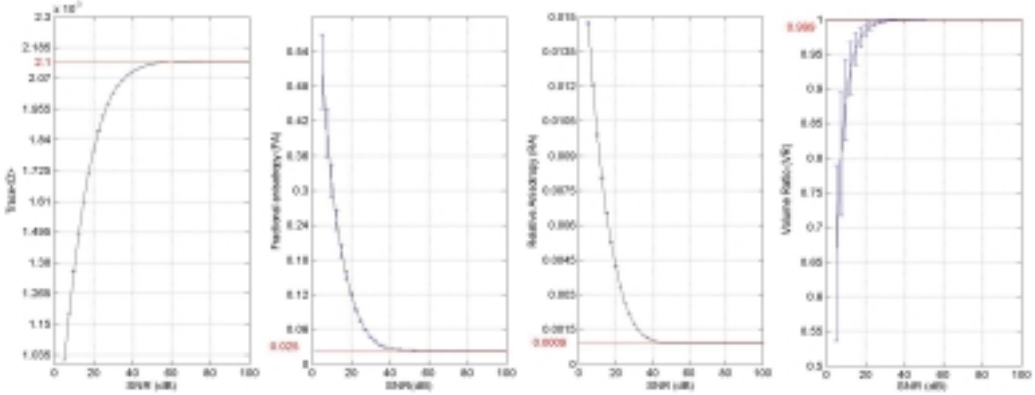
In case of white matter the background noise has an effect on the major Tensor direction,, we study the change in Tensor direction by measuring the mean angle between the real Tensor and the measured one.

III. RESULTS

Figures 2, 3 and 4 show the variation of the calculated means of the Trace <D>, FA, RA and VR ,for the three

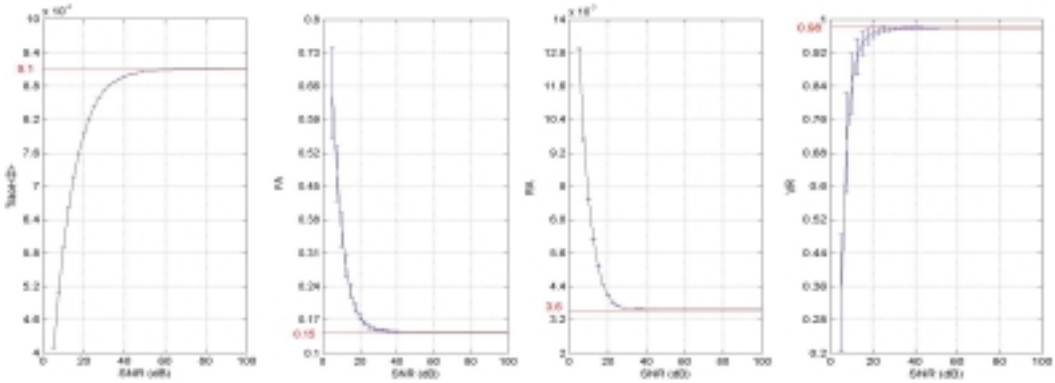
major compartments of the brain (CSF, GM, WM), as a function of the SNR.

The decreasing the SNR causes large divergence on the estimation of all parameters. At low SNR is the max deviation from the real data. For The CSF, the standard deviation at SNR 5 dB is 0.1%for Trace<D>, 12.5% for FA, 0.2 % for RA and 19.5 % for the VR. In case of GM, the deviation 0.1% for Trace <D>, 14.8 % for FA, 0.38 % for RA, 41.78 % for the VR. Finally, for the WM, The deviation 0.1% for Trace <D>, 18.5% for FA, 0.5 % for RA, 80.5 % for the VR.



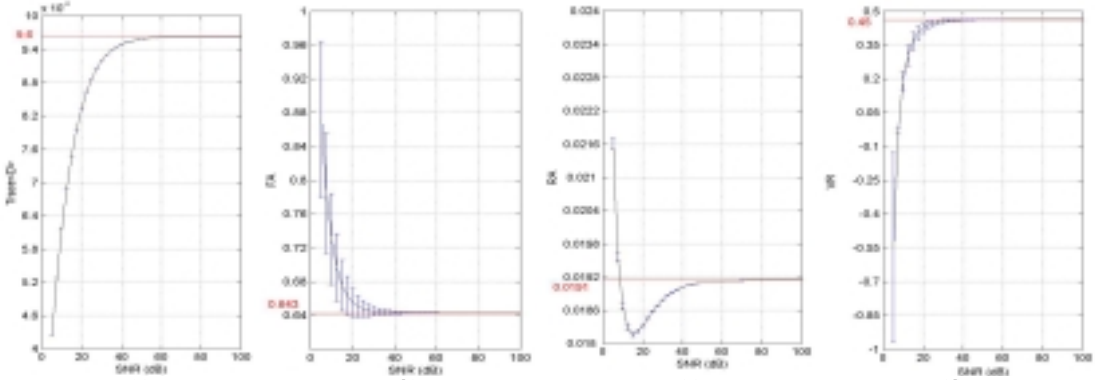
(a) (b) (c) (d)

Fig.2. Simulated values for the CSF, Trace<D>(a),FA(b), RA(c), VR(d).



(a) (b) (c) (d)

Fig.3. Simulated values for the GM, Trace<D>(a),FA(b), RA(c), VR(d).



(a) (b) (c) (d)

Fig.4. Simulated values for the WM, Trace<D>(a),FA(b), RA(c), VR(d).

In case of white matter, the standard deviation of mean cone angle with 44% at 5 dB, which was shown in figure 5. This becomes increasingly pronounced at values of the SNR approximately less than 30 dB.

IV. DISCUSSION

The Trace <D> and the VR were underestimated, but the FA and the RA were overestimated. From the Results we can find that the FA has the highest bias and the VR has

the highest standard deviation at low SNR. The noise performance of the RA is intermediate in the low anisotropy range and worst in the high anisotropy Results from the computer simulations indicate that the probability of obtaining more deviated values for the Anisotropy increases with the increasing of noise level and the anisotropy. Isotropic structures can appear anisotropic and structures with low anisotropy depict higher degree of anisotropy. In practical anisotropy mapping it is, therefore, desirable to optimize SNR. Besides hardware improvements, it is preferable to use diffusion tensor imaging schemes that are less sensitive error propagation, which agrees with Basser *et al.* [5,12].

V. Conclusions

The Monte Carlo simulations presented in this paper suggest that results obtained from DTI investigations will only be meaningful at SNRs greater than approximately 30 dB. At smaller values of the SNR the diffusion anisotropy measured may be significantly overestimated. This overestimation affects both homogeneous and inhomogeneous media alike, so that even isotropic tissue is assigned a high degree of anisotropy.

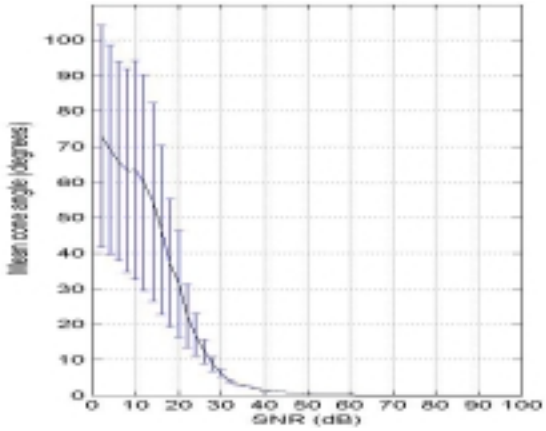


Fig 5. Effect of noise on diffusion Tensor directionality

Appendix I. The simulation procedure

To study the effects of noise level on the accuracy of various diffusion anisotropy indices, we used Monte Carlo simulation. The procedure and related mathematical details are given below.

Step 1. Defining a diagonal matrix, $D_{diag,}$ by assigning typical eigenvalues of the tissue type of interest. to its diagonal elements, where the anisotropy major axis was chosen to be the *z*- axis.

Step 2. The diffusion tensor representation in the laboratory frame, D, is calculated from D_{diag} by an arbitrary rotation. Here, we considered a random 3D rotation.

Step 3. The attenuation at all diffusion gradient direction is computed using b value.

Step 4. The independent white Gaussian noise is generated to be added depending on the SNR,

Step 5. The noise signal is added to the attenuation values, generating the new data.

Step 6. The diffusion tensor is estimated along with its indices including Trace<D>, FA, RA, and VR according to their definitions, for each SNR.

Step 7. The process is repeated a large number of times and the mean and standard deviation of the results are reported.

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