Overview of Chapter 1

- Mathematical representation of signals
- Classification of signals
- Signal manipulation
- Basic signal representation
Classification of Time-Dependent Signals

- Predictability of their behavior
  - Signals can be random or deterministic
- Variation of their time variable and their amplitude
  - Signals can be either continuous-time or discrete-time
  - Signals can be either analog or discrete amplitude, or digital
- Energy content
  - Signals can be characterized as finite- or infinite-energy signals
- Exhibition of repetitive behavior
  - Periodic or aperiodic signals
- Symmetry with respect to the time origin
  - Signals can be even or odd
- Dimension of their support
  - Signals can be of finite or of infinite support. Support
Continuous-Time Signals

- Continuous-amplitude, continuous-time signals are called *analog signals*
- Continuous-amplitude, discrete-time signal is called a *discrete-time signal*
- Discrete-amplitude, discrete-time signal is called a *digital signal*
- If samples of a digital signal are given as binary values, signal is called a *binary signal*
Continuous-Time Signals

- Conversion from continuous to discrete time: *Sampling*
- Conversion from continuous to discrete amplitude: *Quantization or Coding*
Continuous-Time Signals

- Example: Speech Signal

![Analog Signal](image)

- Sampling
- Quantization
- Error
Continuous-Time Signals: Examples

- **Example 1**: \( x(t) = \sqrt{2} \cos(\pi t/2 + \pi/4) \quad -\infty < t < \infty \)
  - Deterministic, analog, periodic, odd, infinite support/energy
- **Example 2**: \( y(t) = (1 + j)e^{j\pi t/2} \quad 0 \leq t \leq 10 \)
  - Deterministic, analog, finite support
- **Example 3**: \( p(t) = 1 \quad 0 \leq t \leq 10 \)
  - Deterministic, analog, finite support
Basic Signal Operations

- Signal addition
- Constant multiplication
- Time and frequency shifting
  - Shift in time: *Delay*
  - Shift in frequency: *Modulation*
- Time scaling
  - Example: $x(-t)$ is a “reflection” of $x(t)$
- Time windowing
  - Multiplication by a window signal $w(t)$
Basic Signal Operations

• Example:
  • (a) original signal
  • (b) delayed version
  • (c) advanced version
  • (d) Reflected version

• Remark:
  ▫ Whenever we combine the delaying or advancing with reflection, delaying and advancing are swapped
  ▫ Ex 1: \(x(-t+1)\) is reflected and delayed
  ▫ Ex 2: \(x(-t-1)\) is reflected and advanced
Basic Signal Operations

Example: Find mathematical expressions for $x(t)$ delayed by 2, advanced by 2, and reflected when:

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- For delay by 2, replace $t$ by $t-2$
  $$x(t - 2) = \begin{cases} 1 & 0 \leq t - 2 \leq 1 \text{ or } 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- For advance by 2
  $$x(t + 2) = \begin{cases} 1 & 0 \leq t + 2 \leq 1 \text{ or } -2 \leq t \leq -1 \\ 0 & \text{otherwise} \end{cases}$$

- For reflection
  $$x(-t) = \begin{cases} 1 & 0 \leq -t \leq 1 \text{ or } -1 \leq t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$
Even and Odd Signals

• Symmetry with respect to the origin
  \[ x(t) \text{ even} : \quad x(t) = x(-t) \]
  \[ x(t) \text{ odd} : \quad x(t) = -x(-t) \]

• Decomposition of any signal as even/odd parts
  \[ y(t) = y_e(t) + y_o(t) \]
  \[ y_e(t) = 0.5 \left[ y(t) + y(-t) \right] \]
  \[ y_o(t) = 0.5 \left[ y(t) - y(-t) \right] \]

• Example: \[ x(t) = \cos(2\pi t + \theta) \quad -\infty < t < \infty \]
  ▫ Neither even nor odd for \( \theta \neq 0 \) or multiples of \( \pi/2 \)
Periodic and Aperiodic Signals

• Analog signal $x(t)$ is periodic if
  ▫ It is defined for all possible values of $t$, $-\infty < t < \infty$
  ▫ there is a positive real value $T_o$, called the period, such that for some integer $k$, $x(t+kT_o) = x(t)$
• The period is the smallest possible value of $T_o > 0$ that makes the periodicity possible.
  ▫ Although $NT_o$ for an integer $N > 1$ is also a period of $x(t)$, it should not be considered *the* period
    • Example: $\cos(2\pi t)$ has a period of 1 not 2 or 3
Periodic and Aperiodic Signals

- Analog sinusoids of frequency $\Omega_0 > 0$ are periodic of period $T_0 = \frac{2\pi}{\Omega_0}$.
  - If $\Omega_0 = 0$, the period is not well defined.
- The sum of two periodic signals $x(t)$ and $y(t)$, of periods $T_1$ and $T_2$, is periodic if the ratio of the periods $T_1/T_2$ is a rational number $N/M$, with $N$ and $M$ being nondivisible.
  - The period of the sum is $MT_1 = NT_2$
- The product of two periodic signals is not necessarily periodic
  - The product of two sinusoids is periodic.
Periodic and Aperiodic Signals

- **Example 1**

Consider a periodic signal \( x(t) \) of period \( T_0 \). Determine whether the following signals are periodic, and if so, find their corresponding periods:

\begin{align*}
(a) \quad & y(t) = A + x(t). \\
(b) \quad & z(t) = x(t) + v(t) \text{ where } v(t) \text{ is periodic of period } T_1 = NT_0, \text{ where } N \text{ is a positive integer.} \\
(c) \quad & w(t) = x(t) + u(t) \text{ where } u(t) \text{ is periodic of period } T_1, \text{ not necessarily a multiple of } T_0. \text{ Determine under what conditions } w(t) \text{ could be periodic.}
\end{align*}

- **Example 2**

Let \( x(t) = e^{j2t} \) and \( y(t) = e^{j\pi t} \), and consider their sum \( z(t) = x(t) + y(t) \), and their product \( w(t) = x(t)y(t) \). Determine if \( z(t) \) and \( w(t) \) are periodic, and if so, find their periods. Is \( p(t) = (1 + x(t))(1 + y(t)) \) periodic?
Finite-Energy and Finite Power Signals

- Concepts of energy and power introduced in circuit theory can be extended to any signal
  - Instantaneous power
  - Energy
  - Power

\[
p(t) = v(t)i(t) = i^2(t) = v^2(t)
\]

\[
E_T = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} i^2(t)dt = \int_{t_0}^{t_1} v^2(t)dt
\]

\[
P_T = \frac{E_T}{T} = \frac{1}{T} \int_{t_0}^{t_1} i^2(t)dt = \frac{1}{T} \int_{t_0}^{t_1} v^2(t)dt
\]
Finite-Energy and Finite Power Signals

• Energy of an analog signal $x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

• Power of an analog signal $x(t)$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

• Signal is **finite energy** (or **square integrable**) if $E_x < \infty$

• Signal is **finite power** if $P_x < \infty$
Finite-Energy and Finite Power Signals: Example

Find the energy and the power of the following:

(a) The periodic signal \( x(t) = \cos(\pi t/2 + \pi/4) \).
(b) The complex signal \( \gamma(t) = (1 + j)e^{j\pi t/2} \), for \( 0 \leq t \leq 10 \) and zero otherwise.
(c) The pulse \( z(t) = 1 \), for \( 0 \leq t \leq 10 \) and zero otherwise.

\[
E_x = \int_{-\infty}^{\infty} \cos^2(\pi t/2 + \pi/4)dt \to \infty
\]

\[
P_x = \frac{1}{8} \int_{0}^{4} \cos(\pi t + \pi/2)dt + \frac{1}{8} \int_{0}^{4} dt = 0 + 0.5 = 0.5
\]

\[
E_y = \int_{0}^{10} |(1 + j)e^{j\pi t/2}|^2 dt = 2 \int_{0}^{10} dt = 20
\]

\[
E_z = \int_{0}^{10} dt = 10
\]

Finite Energy Signals: Zero Power

\[
P_x = \lim_{T \to \infty} \frac{E_x}{2T} = 0
\]
A fundamental idea in signal processing is to attempt to represent signals in terms of basic signals, which we know how to process:

- Complex exponentials
- Sinusoids
- Impulse
- Unit-step
- Ramp
Complex Exponentials

A complex exponential is a signal of the form

\[ x(t) = Ae^{at} \]

\[ = |A|e^{rt} \left[ \cos(\Omega_0 t + \theta) + j \sin(\Omega_0 t + \theta) \right] \quad -\infty < t < \infty \]

where \( A = |A|e^{j\theta} \), and \( a = r + j\Omega_0 \) are complex numbers.

- Depending on the values of \( A \) and \( a \), several signals can be obtained from the complex exponential.
Sinusoids

Sinusoids are of the general form

\[ A \cos(\Omega_0 t + \theta) = A \sin(\Omega_0 t + \theta + \pi/2) \quad -\infty < t < \infty \]

\[ \Omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \]

- Modulation (communication systems)
  - Amplitude modulation (AM)—Amplitude \( A(t) \)
  - Frequency modulation (FM)—Frequency \( \Omega(t) \)
  - Phase modulation (PM)—Phase \( \theta(t) \)
Unit-Step and Unit-Impulse Signals

• The impulse signal $\delta(t)$ is:
  ▫ Zero everywhere except at the origin where its value is not well defined
  ▫ Its area is equal to unity

• Impulse signal $\delta(t)$ and unit step signal $u(t)$ are related by:

\[
\delta(t) = \frac{du(t)}{dt}
\]
Ramp Signal

The ramp signal is defined as

\[ r(t) = t \, u(t) \]

Its relation to the unit-step and the unit-impulse signals is

\[ \frac{dr(t)}{dt} = u(t) \]

\[ \frac{d^2 r(t)}{dt^2} = \delta(t) \]
Basic Signal Operations—Time Scaling, Frequency Shifting, and Windowing

Given a signal \( x(t) \), and real values \( \alpha \neq 0 \) or \( 1 \), and \( \phi > 0 \):

- \( x(\alpha t) \) is said to be contracted if \( |\alpha| > 1 \), and if \( \alpha < 0 \) it is also reflected.
- \( x(\alpha t) \) is said to be expanded if \( |\alpha| < 1 \), and if \( \alpha < 0 \) it is also reflected.
- \( x(t)e^{j\phi t} \) is said to be shifted in frequency by \( \phi \) radians.
- For a window signal \( w(t) \), \( x(t)w(t) \) displays \( x(t) \) within the support of \( w(t) \).
Sifting Property

- Property of the impulse function

\[
\int_{-\infty}^{\infty} f(t) \delta(t - \tau) \, dt = \int_{-\infty}^{\infty} f(\tau) \delta(t - \tau) \, dt = f(\tau) \int_{-\infty}^{\infty} \delta(t - \tau) \, dt = f(\tau) \quad \text{for any } \tau
\]
Problem Assignments

• Problems: 1.4, 1.5, 1.12, 1.13, 1.14
• Partial Solutions available from the student section of the textbook web site